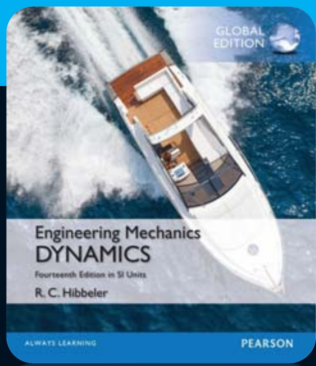
Engineering Mechanics: *Dynamics* in SI Units, 14e



Chapter 12

Kinematics of a Particle

(© Lars Johansson/Fotolia)

12_COC01

Although each of these boats is rather large, from a distance their motion can be analyzed as if each were a particle.

Chapter Objectives

- To introduce the concepts of position, displacement, velocity, and acceleration
- To study particle motion along a straight line and represent this motion graphically
- To investigate particle motion along a curved path using different coordinate systems
- To present an analysis of dependent motion of two particles
- To examine the principles of relative motion of two particles using translating axes

12.1 Introduction

- Mechanics – the state of rest or motion of bodies subjected to the action of forces
- Static – the equilibrium of a body that is either at rest or moves with constant velocity
- Dynamics – deals with accelerated motion of a body

  1) Kinematics – geometric aspects of the motion

  2) Kinetics – analysis of the forces causing the motion

12.2 Rectilinear Kinematics: Continuous Motion

- Rectilinear Kinematics – specifying the particle's position, velocity, and acceleration at any instant
- Position

  1) Single coordinate axis, *s*

  2) Origin, *O*

12.2 Rectilinear Kinematics: Continuous Motion

3) Algebraic Scalar *s* in meters

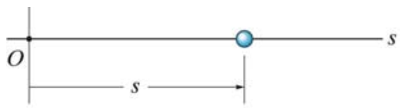


Fig. 12-1 (a)

Note : a. Magnitude of *s* = Dist. from *O* to the particle

b. Direction is defined by algebraic sign on *s*

→ positive = right of the origin

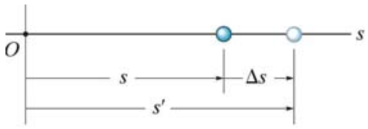
→ negative = left of the origin

12.2 Rectilinear Kinematics: Continuous Motion

Displacement

- Change in its position
- If the particle moves from one point to another, the displacement is :

$$\Delta s = s' - s$$



Displacement

Fig. 12-1 (b)

When Δs is positive / negative,

→ particle's finial position is right / left of its initial position

12.2 Rectilinear Kinematics: Continuous Motion

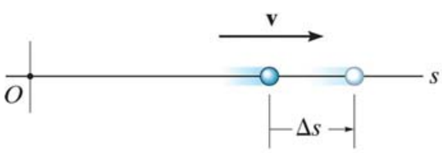
Velocity

- *Average velocity*, $v_{avg} = \dfrac{\Delta s}{\Delta t}$
- *Instantaneous velocity* is defined as

$$v = \lim_{\Delta t \to 0} (\Delta s / \Delta t)$$

or

$$v = \frac{ds}{dt} \quad \text{(12-1)}$$



Velocity

Fig. 12-1 (c)

12.2 Rectilinear Kinematics: Continuous Motion

Velocity

- Magnitude of the velocity is the *speed* (m/s)
- *Average speed* is the total distance traveled by a particle, s_T, divided by the elapsed time Δt .

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t}$$

- The particle travels along the path of length s_T in time Δt

Average speed → $(v_{sp})_{avg} = s_T / \Delta t$

Average velocity → $v_{avg} = \Delta s / \Delta t$

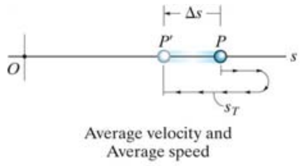


Fig. 12-1 (d)

12.2 Rectilinear Kinematics: Continuous Motion

Acceleration

- Average acceleration is $a_{avg} = \frac{\Delta v}{\Delta t}$

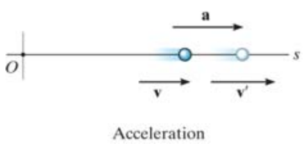


Fig. 12-1 (e)

- Δv represents the difference in the velocity during the time interval Δt, ie $\Delta v = v' - v$
- Instantaneous acceleration is $a = \lim_{\Delta t \to 0} (\Delta v / \Delta t)$

or $a = \frac{dv}{dt}$ (12-2) substituting Eq. 12-1 → $a = \frac{d^2 s}{dt^2}$

12.2 Rectilinear Kinematics: Continuous Motion

Acceleration

- When particle is *slowing down*, its speed is *decreasing* → *decelerating* → $\Delta v = v' - v$ will be negative.
- It will act to the *left*, in the *opposite sense* to *v*
- If the *velocity* is *constant,* the *acceleration is zero.*
- Relation involving the displacement, velocity, and acceleration along the path $dt = \frac{ds}{v} = \frac{dv}{a}$

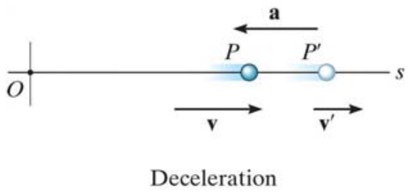


Fig. 12-1 (f)

$$a\,ds = v\,dv \quad (12\text{-}3)$$

12.2 Rectilinear Kinematics: Continuous Motion

Constant acceleration , $a = a_c$.

- Three kinematic equations, $a_c = dv / dt$, $v = ds / dt$, and $a_c\, ds = v\, dv$.

Velocity as a Function of Time

- Integrate $a_c = dv / dt$, assuming that initially $v = v_0$ when $t = 0$.

$$\int_{v_0}^{v} dv = \int_0^t a_c\, dt$$

$$v = v_0 + a_c t$$

Constant Acceleration (12-4)

12.2 Rectilinear Kinematics: Continuous Motion

When the ball is released, it has zero velocity but an acceleration of 9.81 m/s^2.

12.2 Rectilinear Kinematics: Continuous Motion

Position as a Function of Time

- Integrate $v = ds / dt = v_0 + a_c t$, assuming that initially $s = s_0$ when $t = 0$.

$$\int_{s_0}^{s} ds = \int_{0}^{t} (v_0 + a_c t)\, dt$$

$$s = s_0 + v_0 t + \frac{1}{2} a_c t^2$$

Constant Acceleration (12-5)

Velocity as a Function of Position

- Integrate $v\, dv = a_c\, ds$, assuming that initially $v = v_0$ at $s = s_0$

$$\int_{v_0}^{v} v\, dv = \int_{s_0}^{s} a_c\, ds$$

$$v^2 = v_0^2 + 2a_c(s - s_0)$$

Constant Acceleration (12-6)

12.2 Rectilinear Kinematics: Continuous Motion

During the time this rocket undergoes rectilinear motion, its altitude as a function of time can be measured and expressed as $s = s(t)$. Its velocity can then be found using $v = ds / dt$, and its acceleration can be determined from $a = dv / dt$.

12.2 Rectilinear Kinematics: Continuous Motion

Procedure for Analysis

Coordinate System.

- Establish a position coordinate s along the path and specify its *fixed origin* and positive direction.
- Since motion is along a straight line, the vector quantities position, velocity, and acceleration can be represented as algebraic scalars. For analytical work the sense of s, v, and a is then defined by their *algebraic signs*.
- The positive sense for each of these scalars can be indicated by an arrow shown alongside each kinematic equation as it is applied.

Kinematic Equations.

- If a relation is known between any *two* of the four variables a, v, s, and t, then a third variable can be obtained by using one of the kinematic equations, $a = dv/dt$, $v = ds/dt$ or $a\,ds = v\,dv$, since each equation relates all three variables.*
- Whenever integration is performed, it is important that the position and velocity be known at a given instant in order to evaluate either the constant of integration if an indefinite integral is used, or the limits of integration if a definite integral is used.
- Remember that Eqs. 12–4 through 12–6 have only limited use. These equations apply *only* when the *acceleration is constant* and the initial conditions are $s = s_0$ and $v = v_0$ when $t = 0$.

*Some standard differentiation and integration formulas are given in Appendix A.

EXAMPLE 12.1

The car on the left in the photo and in Fig. 12–2 moves in a straight line such that for a short time its velocity is defined by $v = (0.6t^2 + t)$ m/s, where t is in seconds. Determine its position and acceleration when $t = 3$ s. When $t = 0$, $s = 0$.

Fig. 12–2

EXAMPLE 12.1 CONTINUED

SOLUTION

Coordinate System. The position coordinate extends from the fixed origin O to the car, positive to the right.

Position. Since $v = f(t)$, the car's position can be determined from $v = ds/dt$, since this equation relates v, s, and t. Noting that $s = 0$ when $t = 0$, we have*

$$(\overset{+}{\rightarrow}) \qquad v = \frac{ds}{dt} = (0.6t^2 + t)$$

$$\int_0^s ds = \int_0^t (0.6t^2 + t)dt$$

$$s\Big|_0^s = 0.2t^3 + 0.5t^2\Big|_0^t$$

$$s = (0.2t^3 + 0.5t^2) \text{ m}$$

When $t = 3$ s,

$$s = 0.2(3)^3 + 0.5(3)^2 = 9.90 \text{ m} \qquad \textit{Ans.}$$

EXAMPLE 12.1 CONTINUED

Acceleration. Since $v = f(t)$, the acceleration is determined from $a = dv/dt$, since this equation relates a, v, and t.

$$(\overset{+}{\rightarrow}) \qquad a = \frac{dv}{dt} = \frac{d}{dt}(0.6t^2 + t)$$
$$= (1.2t + 1)\ \text{m/s}^2$$

When $t = 3$ s,

$$a = 1.2(3) + 1 = 4.60\ \text{m/s}^2 \rightarrow \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration *cannot* be used to solve this problem, because the acceleration is a function of time.

*The *same result* can be obtained by evaluating a constant of integration C rather than using definite limits on the integral. For example, integrating $ds = (0.6t^2 + t)dt$ yields $s = 0.2t^3 + 0.5t^2 + C$. Using the condition that at $t = 0$, $s = 0$, then $C = 0$.

EXAMPLE 12.2

A small projectile is fired vertically *downward* into a fluid medium with an initial velocity of 60 m/s. Due to the drag resistance of the fluid the projectile experiences a deceleration of $a = (-0.4v^3)\ \text{m/s}^2$, where v is in m/s. Determine the projectile's velocity and position 4 s after it is fired.

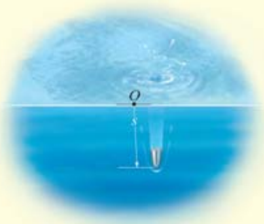


Fig. 12–3

EXAMPLE 12.2 CONTINUED

SOLUTION

Coordinate System. Since the motion is downward, the position coordinate is positive downward, with origin located at O, Fig. 12–3.

Velocity. Here $a = f(v)$ and so we must determine the velocity as a function of time using $a = dv/dt$, since this equation relates v, a, and t. (Why not use $v = v_0 + a_c t$?) Separating the variables and integrating, with $v_0 = 60$ m/s when $t = 0$, yields

$$(+\downarrow) \qquad a = \frac{dv}{dt} = -0.4v^3$$

$$\int_{60\text{ m/s}}^{v} \frac{dv}{-0.4v^3} = \int_0^t dt$$

$$\frac{1}{-0.4}\left(\frac{1}{-2}\right)\frac{1}{v^2}\Bigg|_{60}^{v} = t - 0$$

$$\frac{1}{0.8}\left[\frac{1}{v^2} - \frac{1}{(60)^2}\right] = t$$

$$v = \left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}\right\}\text{ m/s}$$

Here the positive root is taken, since the projectile will continue to move downward. When $t = 4$ s,

$$v = 0.559\text{ m/s}\downarrow \qquad \textit{Ans.}$$

 Gau Lih Book Co., Ltd.

EXAMPLE 12.2 CONTINUED

Position. Knowing $v = f(t)$, we can obtain the projectile's position from $v = ds/dt$, since this equation relates s, v, and t. Using the initial condition $s = 0$, when $t = 0$, we have

$$(+\downarrow) \qquad v = \frac{ds}{dt} = \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2}$$

$$\int_0^s ds = \int_0^t \left[\frac{1}{(60)^2} + 0.8t\right]^{-1/2} dt$$

$$s = \frac{2}{0.8}\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2}\Bigg|_0^t$$

$$s = \frac{1}{0.4}\left\{\left[\frac{1}{(60)^2} + 0.8t\right]^{1/2} - \frac{1}{60}\right\}\text{ m}$$

When $t = 4$ s,

$$s = 4.43\text{ m} \qquad \textit{Ans.}$$

 Gau Lih Book Co., Ltd.

EXAMPLE 12.3

During a test a rocket travels upward at 75 m/s, and when it is 40 m from the ground its engine fails. Determine the maximum height s_B reached by the rocket and its speed just before it hits the ground. While in motion the rocket is subjected to a constant downward acceleration of 9.81 m/s^2 due to gravity. Neglect the effect of air resistance.

SOLUTION

Coordinate System. The origin O for the position coordinate s is taken at ground level with positive upward, Fig. 12–4.

Maximum Height. Since the rocket is traveling *upward*, $v_A = +75$ m/s when $t = 0$. At the maximum height $s = s_B$ the velocity $v_B = 0$. For the entire motion, the acceleration is $a_c = -9.81$ m/s^2 (negative since it acts in the *opposite* sense to positive velocity or positive displacement). Since a_c is *constant* the rocket's position may be related to its velocity at the two points A and B on the path by using Eq. 12–6, namely,

$$(+\uparrow) \qquad v_B^2 = v_A^2 + 2a_c(s_B - s_A)$$
$$0 = (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(s_B - 40 \text{ m})$$
$$s_B = 327 \text{ m} \qquad \textit{Ans.}$$

EXAMPLE 12.3 CONTINUED

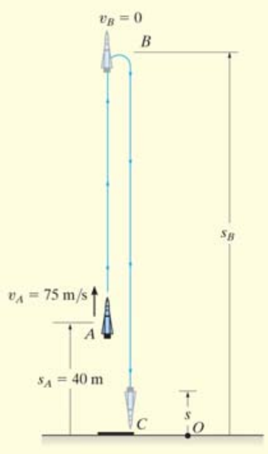


Fig. 12–4

Velocity. To obtain the velocity of the rocket just before it hits the ground, we can apply Eq. 12–6 between points B and C, Fig. 12–4.

$$(+\uparrow) \qquad v_C^2 = v_B^2 + 2a_c(s_C - s_B)$$
$$= 0 + 2(-9.81 \text{ m/s}^2)(0 - 327 \text{ m})$$
$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

The negative root was chosen since the rocket is moving downward. Similarly, Eq. 12–6 may also be applied between points A and C, i.e.,

$$(+\uparrow) \qquad v_C^2 = v_A^2 + 2a_c(s_C - s_A)$$
$$= (75 \text{ m/s})^2 + 2(-9.81 \text{ m/s}^2)(0 - 40 \text{ m})$$
$$v_C = -80.1 \text{ m/s} = 80.1 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

NOTE: It should be realized that the rocket is subjected to a *deceleration* from A to B of 9.81 m/s^2, and then from B to C it is *accelerated* at this rate. Furthermore, even though the rocket momentarily comes to *rest* at B ($v_B = 0$) the acceleration at B is still 9.81 m/s^2 downward!

Copyright © 2017 Pearson Education
Gau Lih Book Co., Ltd.

EXAMPLE 12.4

A metallic particle is subjected to the influence of a magnetic field as it travels downward through a fluid that extends from plate A to plate B, Fig. 12–5. If the particle is released from rest at the midpoint C, $s = 100$ mm, and the acceleration is $a = (4s)$ m/s^2, where s is in meters, determine the velocity of the particle when it reaches plate B, $s = 200$ mm, and the time it takes to travel from C to B.

EXAMPLE 12.4 CONTINUED

SOLUTION

Coordinate System. As shown in Fig. 12–5, s is positive downward, measured from plate A.

Velocity. Since $a = f(s)$, the velocity as a function of position can be obtained by using $v\,dv = a\,ds$. Realizing that $v = 0$ at $s = 0.1$ m, we have

$$(+\downarrow) \qquad v\,dv = a\,ds$$

$$\int_0^v v\,dv = \int_{0.1\text{ m}}^s 4s\,ds$$

$$\frac{1}{2}v^2\Big|_0^v = \frac{4}{2}s^2\Big|_{0.1\text{ m}}^s$$

$$v = 2(s^2 - 0.01)^{1/2}\text{ m/s} \qquad (1)$$

At $s = 200$ mm $= 0.2$ m,

$$v_B = 0.346\text{ m/s} = 346\text{ mm/s}\downarrow \qquad \textit{Ans.}$$

The positive root is chosen since the particle is traveling downward, i.e., in the $+s$ direction.

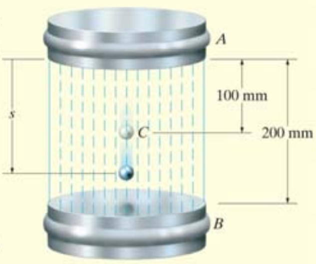


Fig. 12–5

EXAMPLE 12.4 CONTINUED

Time. The time for the particle to travel from C to B can be obtained using $v = ds/dt$ and Eq. 1, where $s = 0.1$ m when $t = 0$. From Appendix A,

$(+\downarrow)$
$$ds = v\,dt$$
$$= 2(s^2 - 0.01)^{1/2}dt$$
$$\int_{0.1}^{s} \frac{ds}{(s^2 - 0.01)^{1/2}} = \int_0^t 2\,dt$$
$$\ln\left(\sqrt{s^2 - 0.01} + s\right)\Big|_{0.1}^{s} = 2t\Big|_0^t$$
$$\ln\left(\sqrt{s^2 - 0.01} + s\right) + 2.303 = 2t$$

At $s = 0.2$ m,

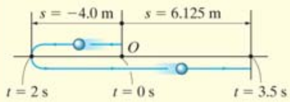
$$t = \frac{\ln\left(\sqrt{(0.2)^2 - 0.01} + 0.2\right) + 2.303}{2} = 0.658 \text{ s} \qquad \textit{Ans.}$$

NOTE: The formulas for constant acceleration cannot be used here because the acceleration changes with position, i.e., $a = 4s$.

 Gau Lih Book Co., Ltd.

EXAMPLE 12.5

A particle moves along a horizontal path with a velocity of $v = (3t^2 - 6t)$ m/s, where t is the time in seconds. If it is initially located at the origin O, determine the distance traveled in 3.5 s, and the particle's average velocity and average speed during the time interval.



(a)

SOLUTION

Coordinate System. Here positive motion is to the right, measured from the origin O, Fig. 12–6a.

Distance Traveled. Since $v = f(t)$, the position as a function of time may be found by integrating $v = ds/dt$ with $t = 0$, $s = 0$.

$(\overset{+}{\rightarrow})$
$$ds = v\,dt$$
$$= (3t^2 - 6t)\,dt$$
$$\int_0^s ds = \int_0^t (3t^2 - 6t)\,dt$$
$$s = (t^3 - 3t^2) \text{ m} \qquad (1)$$

Copyright © 2017 Pearson Education Gau Lih Book Co., Ltd.

EXAMPLE 12.5 CONTINUED

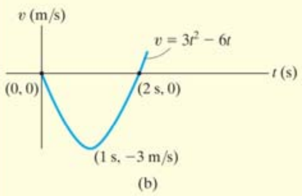


Fig. 12–6

In order to determine the distance traveled in 3.5 s, it is necessary to investigate the path of motion. If we consider a graph of the velocity function, Fig. 12–6*b*, then it reveals that for $0 < t < 2$ s the velocity is *negative*, which means the particle is traveling to the *left*, and for $t > 2$ s the velocity is *positive*, and hence the particle is traveling to the *right*. Also, note that $v = 0$ at $t = 2$ s. The particle's position when $t = 0$, $t = 2$ s, and $t = 3.5$ s can be determined from Eq. 1. This yields

$$s|_{t=0} = 0 \quad s|_{t=2\,s} = -4.0 \text{ m} \quad s|_{t=3.5\,s} = 6.125 \text{ m}$$

The path is shown in Fig. 12–6*a*. Hence, the distance traveled in 3.5 s is

$$s_T = 4.0 + 4.0 + 6.125 = 14.125 \text{ m} = 14.1 \text{ m} \qquad \textit{Ans.}$$

EXAMPLE 12.5 CONTINUED

Velocity. The *displacement* from $t = 0$ to $t = 3.5$ s is

$$\Delta s = s|_{t=3.5\,s} - s|_{t=0} = 6.125 \text{ m} - 0 = 6.125 \text{ m}$$

and so the average velocity is

$$v_{avg} = \frac{\Delta s}{\Delta t} = \frac{6.125 \text{ m}}{3.5 \text{ s} - 0} = 1.75 \text{ m/s} \rightarrow \qquad \textit{Ans.}$$

The average speed is defined in terms of the *distance traveled* s_T. This positive scalar is

$$(v_{sp})_{avg} = \frac{s_T}{\Delta t} = \frac{14.125 \text{ m}}{3.5 \text{ s} - 0} = 4.04 \text{ m/s} \qquad \textit{Ans.}$$

NOTE: In this problem, the acceleration is $a = dv/dt = (6t - 6)$ m/s^2, which is not constant.

12.3 Rectilinear Kinematics: Erratic Motion

- When a particle has erratic motion, a series of functions will be required to specify the motion at different intervals.
- A graph is used to described the relationship with any two of the variables: *a, v, s, t*
- We use $v = ds / dt,\ a = dv / dt$ or $a\ ds = v\ dv$

12.3 Rectilinear Kinematics: Erratic Motion

The *s-t*, *v-t* and *a-t* Graphs

- To construct the v-t graph given the s-t graph, $v = ds / dt$ should be used.

$$\frac{ds}{dt} = v$$

Slope of s-t graph = acceleration

- By measuring the slope on the s-t graph when $t = t_1$, the velocity is v_1, the v-t graph can be constructed.

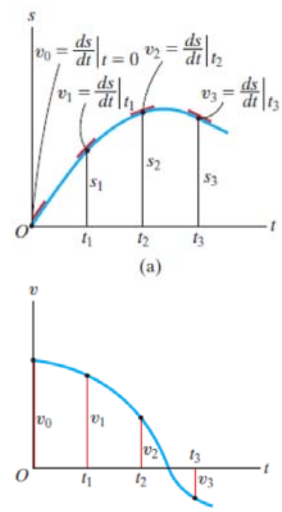


Fig. 12-7 (b)

12.3 Rectilinear Kinematics: Erratic Motion

The *s-t*, *v-t* and *a-t* Graphs

- When the particle's v-t graph is known, the a-t graph can be determined using $a = dv / dt$

$$\frac{dv}{dt} = a$$

Slope of v-t graph = acceleration

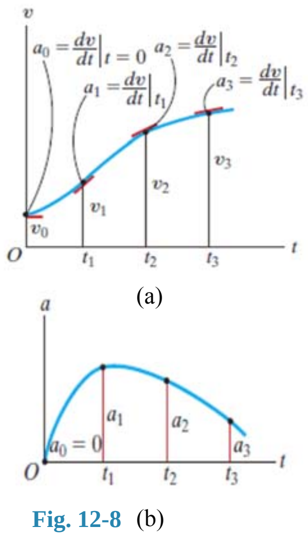


Fig. 12-8

12.3 Rectilinear Kinematics: Erratic Motion

The *s-t*, *v-t* and *a-t* Graphs

- When a–t graph is given, v–t can be written as

$$\Delta v = \int a \ dt$$

change in velocity = area under a-t graph

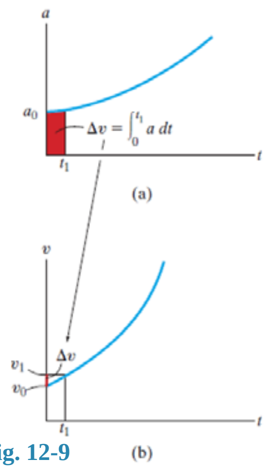


Fig. 12-9

12.3 Rectilinear Kinematics: Erratic Motion

The *s-t*, *v-t* and *a-t* Graphs

- When v–t graph is given, s–t can be written as

$$\Delta s = \int v \, dt$$

displacement = area under v-t graph

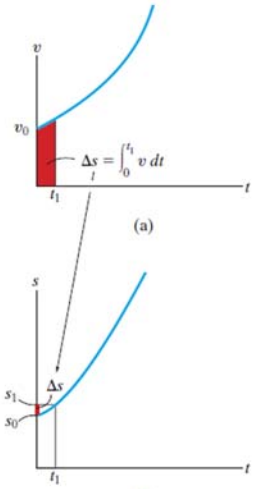


Fig. 12-10

12.3 Rectilinear Kinematics: Erratic Motion

The *v-s* and *a-s* Graphs

- If the a–s graph can be constructed, then we have :

$$\tfrac{1}{2}\left(v_1^2 - v_0^2\right) = \int_{s_0}^{s_1} a \; ds$$

area under a-s graph

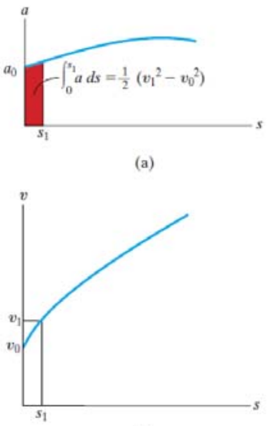


Fig. 12-11

12.3 Rectilinear Kinematics: Erratic Motion

The v-*s* and *a*-*s* Graphs

- When v–s graph is known, *a* at any position *s* can be written as

$$a = v\left(\frac{dv}{ds}\right)$$

Acceleration = velocity times slope of v-s graph

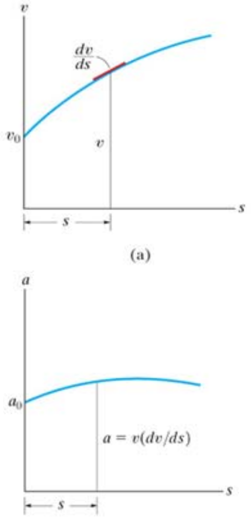
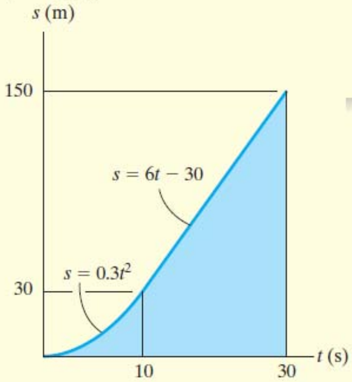


Fig. 12-12

EXAMPLE 12.6

A bicycle moves along a straight road such that its position is described by the graph shown in Fig. 12–13*a*. Construct the v–t and a–t graphs for $0 \le t \le 30$ s.



(a)

EXAMPLE 12.6 CONTINUED

SOLUTION

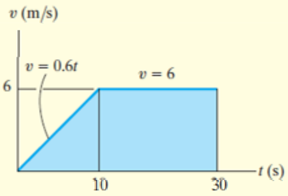
v–t Graph. Since $v = ds/dt$, the v–t graph can be determined by differentiating the equations defining the s–t graph, Fig. 12–13a. We have

$$0 \leq t < 10\text{ s}; \qquad s = (0.3t^2)\text{ m} \qquad v = \frac{ds}{dt} = (0.6t)\text{ m/s}$$

$$10\text{ s} < t \leq 30\text{ s}; \qquad s = (6t - 30)\text{ m} \qquad v = \frac{ds}{dt} = 6\text{ m/s}$$

The results are plotted in Fig. 12–13b. We can also obtain specific values of v by measuring the *slope* of the s–t graph at a given instant. For example, at $t = 20$ s, the slope of the s–t graph is determined from the straight line from 10 s to 30 s, i.e.,

$$t = 20\text{ s}; \qquad v = \frac{\Delta s}{\Delta t} = \frac{150\text{ m} - 30\text{ m}}{30\text{ s} - 10\text{ s}} = 6\text{ m/s}$$



(b)

EXAMPLE 12.6 CONTINUED

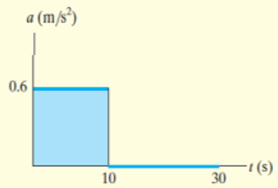
a–t Graph. Since $a = dv/dt$, the a–t graph can be determined by differentiating the equations defining the lines of the v–t graph. This yields

$$0 \leq t < 10\text{ s}; \qquad v = (0.6t)\text{ m/s} \qquad a = \frac{dv}{dt} = 0.6\text{ m/s}^2$$

$$10 < t \leq 30\text{ s}; \qquad v = 6\text{ m/s} \qquad a = \frac{dv}{dt} = 0$$

The results are plotted in Fig. 12–13c.

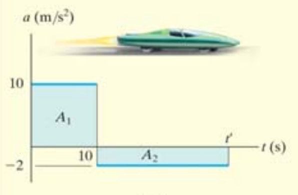
NOTE: Show that $a = 0.6\text{ m/s}^2$ when $t = 5$ s by measuring the slope of the v–t graph.



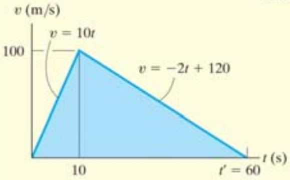
(c)

Fig. 12–13

EXAMPLE 12.7



(a)



(b)

The car in Fig. 12–14*a* starts from rest and travels along a straight track such that it accelerates at $10\ \text{m/s}^2$ for 10 s, and then decelerates at $2\ \text{m/s}^2$. Draw the v–t and s–t graphs and determine the time t' needed to stop the car. How far has the car traveled?

SOLUTION

v–t Graph. Since $dv = a\,dt$, the v–t graph is determined by integrating the straight-line segments of the a–t graph. Using the *initial condition* $v = 0$ when $t = 0$, we have

$$0 \le t < 10\ \text{s};\quad a = (10)\ \text{m/s}^2;\quad \int_0^v dv = \int_0^t 10\,dt,\quad v = 10t$$

When $t = 10$ s, $v = 10(10) = 100$ m/s. Using this as the *initial condition* for the next time period, we have

$$10\ \text{s} < t \le t';\ a = (-2)\ \text{m/s}^2;\ \int_{100\ \text{m/s}}^v dv = \int_{10\ \text{s}}^t -2\,dt,\ v = (-2t + 120)\ \text{m/s}$$

When $t = t'$ we require $v = 0$. This yields, Fig. 12–14*b*,

$$t' = 60\ \text{s} \qquad \textit{Ans.}$$

A more direct solution for t' is possible by realizing that the area under the a–t graph is equal to the change in the car's velocity. We require $\Delta v = 0 = A_1 + A_2$, Fig. 12–14*a*. Thus

$$0 = 10\ \text{m/s}^2(10\ \text{s}) + (-2\ \text{m/s}^2)(t' - 10\ \text{s})$$

$$t' = 60\ \text{s} \qquad \textit{Ans.}$$

EXAMPLE 12.7 CONTINUED

s–t Graph. Since $ds = v\,dt$, integrating the equations of the v–t graph yields the corresponding equations of the s–t graph. Using the *initial condition* $s = 0$ when $t = 0$, we have

$$0 \le t \le 10\ \text{s};\quad v = (10t)\ \text{m/s};\quad \int_0^s ds = \int_0^t 10t\,dt,\quad s = (5t^2)\ \text{m}$$

When $t = 10$ s, $s = 5(10)^2 = 500$ m. Using this *initial condition*,

$$10\ \text{s} \le t \le 60\ \text{s};\ v = (-2t + 120)\ \text{m/s};\ \int_{500\ \text{m}}^s ds = \int_{10\ \text{s}}^t (-2t + 120)\,dt$$

$$s - 500 = -t^2 + 120t - [-(10)^2 + 120(10)]$$

$$s = (-t^2 + 120t - 600)\ \text{m}$$

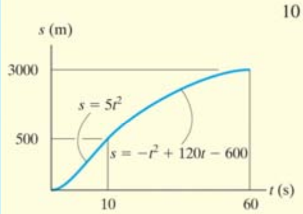
When $t' = 60$ s, the position is

$$s = -(60)^2 + 120(60) - 600 = 3000\ \text{m} \qquad \textit{Ans.}$$

The s–t graph is shown in Fig. 12–14*c*.

NOTE: A direct solution for s is possible when $t' = 60$ s, since the *triangular area* under the v–t graph would yield the displacement $\Delta s = s - 0$ from $t = 0$ to $t' = 60$ s. Hence,

$$\Delta s = \tfrac{1}{2}(60\ \text{s})(100\ \text{m/s}) = 3000\ \text{m} \qquad \textit{Ans.}$$

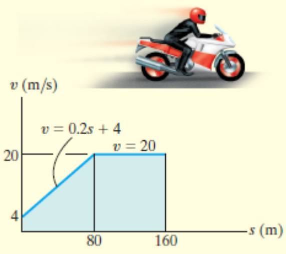


(c)

Fig. 12–14

EXAMPLE 12.8

The v–s graph describing the motion of a motorcycle is shown in Fig. 12–15a. Construct the a–s graph of the motion and determine the time needed for the motorcycle to reach the position $s = 160$ m.



(a)

SOLUTION

a–s Graph. Since the equations for segments of the v–s graph are given, the a–s graph can be determined using $a\,ds = v\,dv$.

$0 \le s < 80$ m; $\quad v = (0.2s + 4)$ m/s

$$a = v\frac{dv}{ds} = (0.2s + 4)\frac{d}{ds}(0.2s + 4) = 0.04s + 0.8$$

$80 \text{ m} < s \le 160$ m; $\quad v = 20$ m/s

$$a = v\frac{dv}{ds} = (20)\frac{d}{ds}(20) = 0$$

The results are plotted in Fig. 12–15b.

EXAMPLE 12.8 CONTINUED

Time. The time can be obtained using the v–s graph and $v = ds/dt$, because this equation relates v, s, and t. For the first segment of motion, $s = 0$ when $t = 0$, so

$0 \le s < 80$ m; $\quad v = (0.2s + 4)$ m/s; $\quad dt = \dfrac{ds}{v} = \dfrac{ds}{0.2s + 4}$

$$\int_0^t dt = \int_0^s \frac{ds}{0.2s + 4}$$

$$t = \left[5 \ln\left(\frac{0.2s + 4}{4}\right)\right] \text{s}$$

At $s = 80$ m, $t = 5 \ln\left[\dfrac{0.2(80) + 4}{4}\right] = 8.047$ s. Therefore, using these initial conditions for the second segment of motion,

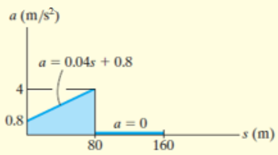
$80 \text{ m} < s \le 160$ m; $\quad v = 20$ m/s; $\quad dt = \dfrac{ds}{v} = \dfrac{ds}{20}$

$$\int_{8.047\text{ s}}^t dt = \int_{80\text{ m}}^s \frac{ds}{20};$$

$$t - 8.047 = \frac{s}{20} - 4; \quad t = \left(\frac{s}{20} + 4.047\right) \text{s}$$

Therefore, at $s = 160$ m,

$$t = \frac{160}{20} + 4.047 = 12.0 \text{ s} \qquad \textit{Ans.}$$



(b)

Fig. 12–15

EXAMPLE 12.8 CONTINUED

At $s = 80$ m, $t = 5 \ln\left[\frac{0.2(80) + 4}{4}\right] = 8.047$ s. Therefore, using these initial conditions for the second segment of motion,

$$80 \text{ m} < s \leq 160 \text{ m}; \qquad v = 20 \text{ m/s}; \qquad dt = \frac{ds}{v} = \frac{ds}{20}$$

$$\int_{8.047\text{ s}}^{t} dt = \int_{80\text{ m}}^{s} \frac{ds}{20};$$

$$t - 8.047 = \frac{s}{20} - 4; \quad t = \left(\frac{s}{20} + 4.047\right) \text{s}$$

Therefore, at $s = 160$ m,

$$t = \frac{160}{20} + 4.047 = 12.0 \text{ s} \qquad \textit{Ans.}$$

NOTE: The graphical results can be checked in part by calculating slopes. For example, at $s = 0$, $a = v(dv/ds) = 4(20 - 4)/80 = 0.8 \text{ m/s}^2$. Also, the results can be checked in part by inspection. The v–s graph indicates the initial increase in velocity (acceleration) followed by constant velocity ($a = 0$).

12.4 General Curvilinear Motion

- Curvilinear motion occurs when a particle moves along a curved path

Position

- measured from a fixed point *O*, by the *position vector* $\mathbf{r} = \mathbf{r}(t)$

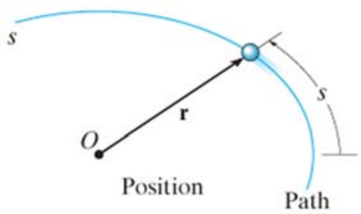


Fig. 12-16 (a)

12.4 General Curvilinear Motion

Displacement

- During a small time interval Δt the particle moves a distance Δs along the curve to a new position, defined by $\mathbf{r'} = \mathbf{r} + \Delta\mathbf{r}$
- The *displacement* $\Delta\mathbf{r}$ represents the change in the particle's position → $\Delta\mathbf{r} = \mathbf{r'} - \mathbf{r}$

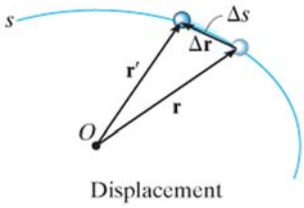


Fig. 12-16 (b)

12.4 General Curvilinear Motion

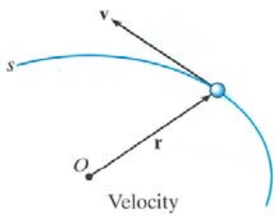


Fig. 12-16 (c)

Velocity

- *Average velocity* of the particle is :

$$v_{avg} = \frac{\Delta \mathbf{r}}{\Delta t}$$

- Instantaneous velocity is determined by letting Δt → 0,

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} \quad (12\text{-}7)$$

- Approaches the arc length Δs as Δt → 0, we have :

$$v = \frac{ds}{dt} \quad (12\text{-}8)$$

12.4 General Curvilinear Motion

Acceleration

- The *average acceleration* during the time interval Δt is

$$\mathbf{a}_{avg} = \frac{\Delta \mathbf{v}}{\Delta t} \qquad \mathbf{a} = \frac{d\mathbf{v}}{dt} \quad \text{(12-9)} \qquad \rightarrow \quad \mathbf{a} = \frac{d^2\mathbf{r}}{dt^2}$$

- ***a*** acts tangent to the *hodograph* and is not tangent to the path of motion

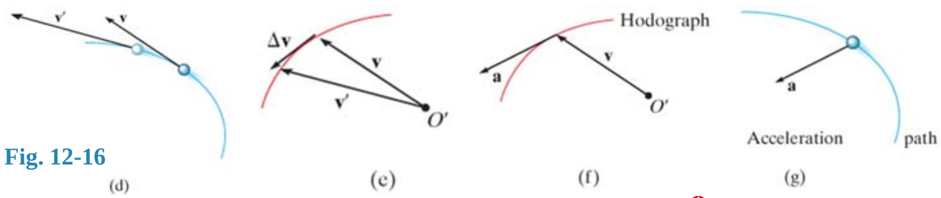


Fig. 12-16

12.5 Curvilinear Motion: Rectangular Components

Position

- Location is defined by the *position vector*

$$\mathbf{r} = x\mathbf{i} + y\mathbf{j} + z\mathbf{k} \quad \text{(12-10)}$$

- The magnitude of **r** is defined as : $r = \sqrt{x^2 + y^2 + z^2}$
- The *direction* of *r* is specified by the unit vector $\mathbf{u}_r = \mathbf{r}/r$.

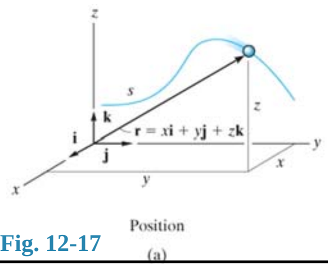


Fig. 12-17

12.5 Curvilinear Motion: Rectangular Components

Velocity

- The first time derivative of $\mathbf{r}$ yields the velocity :

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = \frac{d}{dt}(x\mathbf{i}) + \frac{d}{dt}(y\mathbf{j}) + \frac{d}{dt}(z\mathbf{k})$$

- The derivative of the $\mathbf{i}$ component of $\mathbf{r}$ is :

$$\frac{d}{dt}(x\mathbf{i}) = \frac{dx}{dt}\mathbf{i} + x\frac{d\mathbf{i}}{dt}$$

- The final result :

$$\mathbf{v} = \frac{d\mathbf{r}}{dt} = v_x\mathbf{i} + v_y\mathbf{j} + v_z\mathbf{k} \quad \textbf{(12-11)}$$

where

$$v_x = \dot{x} \quad v_y = \dot{y} \quad v_z = \dot{z} \quad \textbf{(12-12)}$$

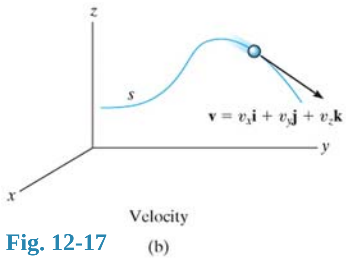


Fig. 12-17

12.5 Curvilinear Motion: Rectangular Components

Velocity

- The velocity has a *magnitude* that is found from :

$$v = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

- has a direction specified by the unit vector $\mathbf{u}_v = \mathbf{v}/v$ and is *always tangent to the path*

12.5 Curvilinear Motion: Rectangular Components

Acceleration

- We have $$a = \frac{d\mathbf{v}}{dt} = a_x\mathbf{i} + a_y\mathbf{j} + a_z\mathbf{k}$$ (12-13)

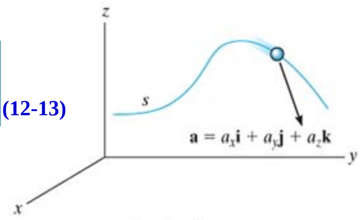


Fig. 12-17 (c)

where

$$a_x = \dot{v}_x = \ddot{x}$$
$$a_y = \dot{v}_y = \ddot{y}$$
$$a_z = \dot{v}_z = \ddot{z}$$
(12-14)

- The acceleration has a *magnitude* :

$$a = \sqrt{a_x^2 + a_y^2 + a_z^2}$$

12.5 Curvilinear Motion: Rectangular Components

Procedure for Analysis

Coordinate System.

- A rectangular coordinate system can be used to solve problems for which the motion can conveniently be expressed in terms of its *x*, *y*, *z* components.

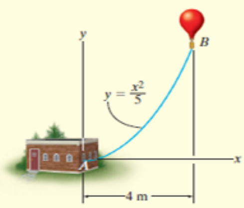
12.5 Curvilinear Motion: Rectangular Components

Kinematic Quantities.

- Since *rectilinear motion* occurs along *each coordinate axis*, the motion along each axis is found using $v = ds/dt$ and $a = dv/dt$; or in cases where the motion is not expressed as a function of time, the equation $a\,ds = v\,dv$ can be used.
- In two dimensions, the equation of the path $y = f(x)$ can be used to relate the x and y components of velocity and acceleration by applying the chain rule of calculus. A review of this concept is given in Appendix C.
- Once the x, y, z components of **v** and **a** have been determined, the magnitudes of these vectors are found from the Pythagorean theorem, Eq. B–3, and their coordinate direction angles from the components of their unit vectors, Eqs. B–4 and B–5.

EXAMPLE 12.9

At any instant the horizontal position of the weather balloon in Fig. 12–18*a* is defined by $x = (2t)$ m, where t is in seconds. If the equation of the path is $y = x^2/5$, determine the magnitude and direction of the velocity and the acceleration when $t = 2$ s.



(a)

SOLUTION

Velocity. The velocity component in the x direction is

$$v_x = \dot{x} = \frac{d}{dt}(2t) = 2\text{ m/s} \rightarrow$$

To find the relationship between the velocity components we will use the chain rule of calculus. When $t = 2$ s, $x = 2(2) = 4$ m, Fig. 12–18*a*, and so

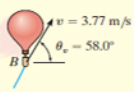
$$v_y = \dot{y} = \frac{d}{dt}(x^2/5) = 2x\dot{x}/5 = 2(4)(2)/5 = 3.20\text{ m/s}\uparrow$$

When $t = 2$ s, the magnitude of velocity is therefore

$$v = \sqrt{(2\text{ m/s})^2 + (3.20\text{ m/s})^2} = 3.77\text{ m/s} \qquad \textit{Ans.}$$

The direction is tangent to the path, Fig. 12–18*b*, where

$$\theta_v = \tan^{-1}\frac{v_y}{v_x} = \tan^{-1}\frac{3.20}{2} = 58.0^\circ \qquad \textit{Ans.}$$



(b)

EXAMPLE 12.9 CONTINUED

Acceleration. The relationship between the acceleration components is determined using the chain rule. (See Appendix C.) We have

$$a_x = \dot{v}_x = \frac{d}{dt}(2) = 0$$

$$a_y = \dot{v}_y = \frac{d}{dt}(2x\dot{x}/5) = 2(\dot{x})\dot{x}/5 + 2x(\ddot{x})/5$$

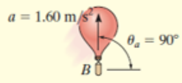
$$= 2(2)^2/5 + 2(4)(0)/5 = 1.60 \text{ m/s}^2 \uparrow$$

Thus,

$$a = \sqrt{(0)^2 + (1.60)^2} = 1.60 \text{ m/s}^2 \qquad \textit{Ans.}$$

The direction of **a**, as shown in Fig. 12–18*c*, is

$$\theta_a = \tan^{-1}\frac{1.60}{0} = 90° \qquad \textit{Ans.}$$



(c)

Fig. 12–18

NOTE: It is also possible to obtain v_y and a_y by first expressing $y = f(t) = (2t)^2/5 = 0.8t^2$ and then taking successive time derivatives.

 Gau Lih Book Co., Ltd.

EXAMPLE 12.10

(© R.C. Hibbeler)

For a short time, the path of the plane in Fig. 12–19*a* is described by $y = (0.001x^2)$ m. If the plane is rising with a constant upward velocity of 10 m/s, determine the magnitudes of the velocity and acceleration of the plane when it reaches an altitude of $y = 100$ m.

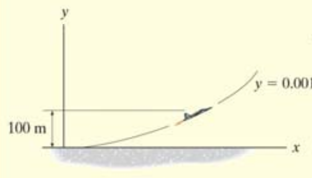
SOLUTION

When $y = 100$ m, then $100 = 0.001x^2$ or $x = 316.2$ m. Also, due to constant velocity $v_y = 10$ m/s, so

$$y = v_y t; \qquad 100 \text{ m} = (10 \text{ m/s})\, t \qquad t = 10 \text{ s}$$

 Gau Lih Book Co., Ltd.

EXAMPLE 12.10 CONTINUED



(a)

Velocity. Using the chain rule (see Appendix C) to find the relationship between the velocity components, we have

$$y = 0.001x^2$$

$$v_y = \dot{y} = \frac{d}{dt}(0.001x^2) = (0.002x)\dot{x} = 0.002xv_x \qquad (1)$$

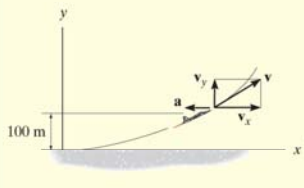
Thus

$$10 \text{ m/s} = 0.002(316.2 \text{ m})(v_x)$$
$$v_x = 15.81 \text{ m/s}$$

The magnitude of the velocity is therefore

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{(15.81 \text{ m/s})^2 + (10 \text{ m/s})^2} = 18.7 \text{ m/s} \qquad \textit{Ans.}$$

EXAMPLE 12.10 CONTINUED



(b)

Fig. 12–19

Acceleration. Using the chain rule, the time derivative of Eq. (1) gives the relation between the acceleration components.

$$a_y = \dot{v}_y = (0.002\dot{x})\dot{x} + 0.002x(\ddot{x}) = 0.002(v_x^2 + xa_x)$$

When $x = 316.2$ m, $v_x = 15.81$ m/s, $\dot{v}_y = a_y = 0$,

$$0 = 0.002\left[(15.81 \text{ m/s})^2 + 316.2 \text{ m}(a_x)\right]$$
$$a_x = -0.791 \text{ m/s}^2$$

The magnitude of the plane's acceleration is therefore

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(-0.791 \text{ m/s}^2)^2 + (0 \text{ m/s}^2)^2}$$
$$= 0.791 \text{ m/s}^2 \qquad \textit{Ans.}$$

These results are shown in Fig. 12–19*b*.

12.6 Motion of Projectile

- Projectile launched at (x_0, y_0)
- Air resistance is neglected
- Only force is its weight downwards
- $a_c = g = 9.81$ m/s

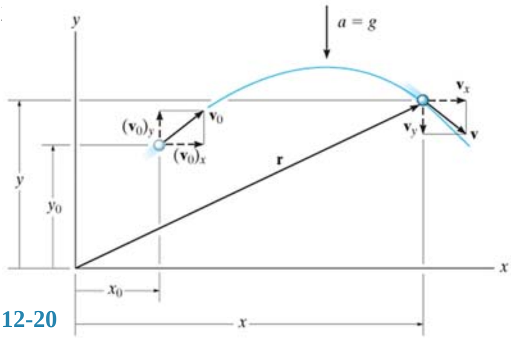
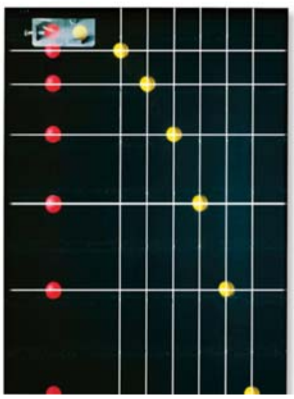


Fig. 12-20

12.6 Motion of Projectile



Each picture in this sequence is taken after the same time interval. The red ball falls from rest, whereas the yellow ball is given a horizontal velocity when released. Both balls accelerate downward at the same rate, and so they remain at the same elevation at any instant.
This acceleration causes the difference in elevation between the balls to increase between successive photos. Also, note the horizontal distance between successive photos of the yellow ball is constant since the velocity in the horizontal direction remains constant.

12.6 Motion of Projectile

Horizontal Motion

- Since $a_x = 0$,

$(\overset{+}{\rightarrow})$ $v = v_0 + a_c t;$ $\qquad v_x = (v_0)_x$

$(\overset{+}{\rightarrow})$ $x = x_0 + v_0 t + \frac{1}{2} a_c t^2;$ $\qquad x = x_0 + (v_0)_x t$

$(\overset{+}{\rightarrow})$ $v^2 = v_0^2 + 2a_c(x - x_0);$ $\qquad v_x = (v_0)_x$

- Horizontal component of velocity always remain constant during the motion

12.6 Motion of Projectile

Vertical Motion

- Positive *y* axis is upward, then $a_y = -g$

$(+\uparrow)$ $v = v_0 + a_c t;$ $\qquad v_y = (v_0)_y - gt$

$(+\uparrow)$ $y = y_0 + v_0 t + \frac{1}{2} a_c t^2;$ $\qquad y = y_0 + (v_0)_y t - \frac{1}{2} g t^2$

$(+\uparrow)$ $v^2 = v_0^2 + 2a_c(y - y_0);$ $\qquad v_y^2 = (v_0)_y^2 - 2g(y - y_0)$

12.6 Motion of Projectile

Procedure for Analysis

Coordinate System.

- Establish the fixed x, y coordinate axes and sketch the trajectory of the particle. Between any *two points* on the path specify the given problem data and identify the *three unknowns*. In all cases the acceleration of gravity acts downward and equals 9.81 m/s^2. The particle's initial and final velocities should be represented in terms of their x and y components.

- Remember that positive and negative position, velocity, and acceleration components always act in accordance with their associated coordinate directions.

12.6

Kinematic Equations.

- Depending upon the known data and what is to be determined, a choice should be made as to which three of the following four equations should be applied between the two points on the path to obtain the most direct solution to the problem.

Horizontal Motion.

- The *velocity* in the horizontal or x direction is *constant*, i.e., $v_x = (v_0)_x$, and

$$x = x_0 + (v_0)_x t$$

Vertical Motion.

- In the vertical or y direction *only two* of the following three equations can be used for solution.

$$v_y = (v_0)_y + a_c t$$

$$y = y_0 + (v_0)_y t + \tfrac{1}{2} a_c t^2$$

$$v_y^2 = (v_0)_y^2 + 2a_c(y - y_0)$$

For example, if the particle's final velocity v_y is not needed, then the first and third of these equations will not be useful.

12.6 Motion of Projectile

Once thrown, the basketball follows a parabolic trajectory.

12.6 Motion of Projectile

Gravel falling off the end of this conveyor belt follows a path that can be predicted using the equations of constant acceleration. In this way the location of the accumulated pile can be determined. Rectangular coordinates are used for the analysis since the acceleration is only in the vertical direction.

EXAMPLE 12.11

A sack slides off the ramp, shown in Fig. 12–21, with a horizontal velocity of 12 m/s. If the height of the ramp is 6 m from the floor, determine the time needed for the sack to strike the floor and the range R where sacks begin to pile up.

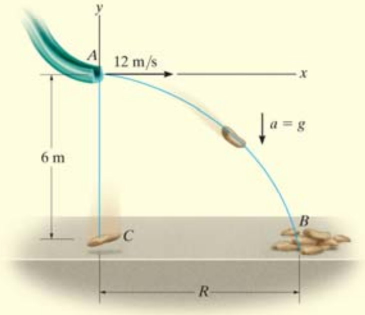


Fig. 12–21

EXAMPLE 12.11 CONTINUED

SOLUTION

Coordinate System. The origin of coordinates is established at the beginning of the path, point A, Fig. 12–21. The initial velocity of a sack has components $(v_A)_x = 12$ m/s and $(v_A)_y = 0$. Also, between points A and B the acceleration is $a_y = -9.81$ m/s^2. Since $(v_B)_x = (v_A)_x = 12$ m/s, the three unknowns are $(v_B)_y$, R, and the time of flight t_{AB}. Here we do not need to determine $(v_B)_y$.

Vertical Motion. The vertical distance from A to B is known, and therefore we can obtain a direct solution for t_{AB} by using the equation

$(+\uparrow)$
$$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2}a_c t_{AB}^2$$
$$-6 \text{ m} = 0 + 0 + \tfrac{1}{2}(-9.81 \text{ m/s}^2)t_{AB}^2$$
$$t_{AB} = 1.11 \text{ s} \qquad \textit{Ans.}$$

Horizontal Motion. Since t_{AB} has been calculated, R is determined as follows:

$(\overset{+}{\rightarrow})$
$$x_B = x_A + (v_A)_x t_{AB}$$
$$R = 0 + 12 \text{ m/s}\,(1.11 \text{ s})$$
$$R = 13.3 \text{ m} \qquad \textit{Ans.}$$

NOTE: The calculation for t_{AB} also indicates that if a sack were released *from rest* at A, it would take the same amount of time to strike the floor at C, Fig. 12–21.

EXAMPLE 12.12

The chipping machine is designed to eject wood chips at $v_O = 7.5$ m/s as shown in Fig. 12–22. If the tube is oriented at 30° from the horizontal, determine how high, h, the chips strike the pile if at this instant they land on the pile 6 m from the tube.

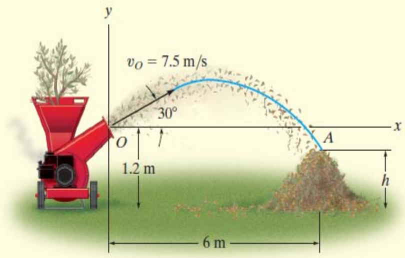


Fig. 12–22

EXAMPLE 12.12 CONTINUED

SOLUTION

Coordinate System. When the motion is analyzed between points O and A, the three unknowns are the height h, time of flight t_{OA}, and vertical component of velocity $(v_A)_y$. [Note that $(v_A)_x = (v_O)_x$.] With the origin of coordinates at O, Fig. 12–22, the initial velocity of a chip has components of

$$(v_O)_x = (7.5 \cos 30°) \text{ m/s} = 6.50 \text{ m/s} \rightarrow$$

$$(v_O)_y = (7.5 \sin 30°) \text{ m/s} = 3.75 \text{ m/s} \uparrow$$

Also, $(v_A)_x = (v_O)_x = 6.50$ m/s and $a_y = -9.81$ m/s^2. Since we do not need to determine $(v_A)_y$, we have

EXAMPLE 12.12 CONTINUED

Horizontal Motion.

$$(\overset{+}{\rightarrow}) \quad x_A = x_O + (v_O)_x t_{OA}$$
$$6\text{ m} = 0 + (6.50\text{ m/s})t_{OA}$$
$$t_{OA} = 0.923\text{ s}$$

Vertical Motion. Relating t_{OA} to the initial and final elevations of a chip, we have

$$(+\uparrow) \quad y_A = y_O + (v_O)_y t_{OA} + \tfrac{1}{2}a_c t_{OA}^2$$
$$(h - 1.2\text{ m}) = 0 + (3.75\text{ m/s})(0.923\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(0.923\text{ s})^2$$
$$h = 0.483\text{ m}$$ *Ans.*

NOTE: We can determine $(v_A)_y$ by using $(v_A)_y = (v_O)_y + a_c t_{OA}$.

EXAMPLE 12.13

The track for this racing event was designed so that riders jump off the slope at 30°, from a height of 1 m. During a race it was observed that the rider shown in Fig. 12–23*a* remained in mid air for 1.5 s. Determine the speed at which he was traveling off the ramp, the horizontal distance he travels before striking the ground, and the maximum height he attains. Neglect the size of the bike and rider.

(© R.C. Hibbeler)

(a)

EXAMPLE 12.13 CONTINUED

SOLUTION

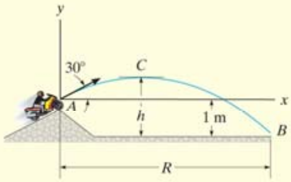
Coordinate System. As shown in Fig. 12–23*b*, the origin of the coordinates is established at A. Between the end points of the path AB the three unknowns are the initial speed v_A, range R, and the vertical component of velocity $(v_B)_y$.

Vertical Motion. Since the time of flight and the vertical distance between the ends of the path are known, we can determine v_A.

$(+\uparrow)$ $$y_B = y_A + (v_A)_y t_{AB} + \tfrac{1}{2} a_c t_{AB}^2$$

$$-1\text{ m} = 0 + v_A \sin 30°(1.5\text{ s}) + \tfrac{1}{2}(-9.81\text{ m/s}^2)(1.5\text{ s})^2$$

$$v_A = 13.38\text{ m/s} = 13.4\text{ m/s} \qquad \textit{Ans.}$$



(b)

Fig. 12–23

EXAMPLE 12.13 CONTINUED

Horizontal Motion. The range R can now be determined.

$(\overset{+}{\rightarrow})$ $$x_B = x_A + (v_A)_x t_{AB}$$

$$R = 0 + 13.38 \cos 30°\text{ m/s}(1.5\text{ s})$$

$$= 17.4\text{ m} \qquad \textit{Ans.}$$

In order to find the maximum height h we will consider the path AC, Fig. 12–23*b*. Here the three unknowns are the time of flight t_{AC}, the horizontal distance from A to C, and the height h. At the maximum height $(v_C)_y = 0$, and since v_A is known, we can determine h *directly* without considering t_{AC} using the following equation.

$$(v_C)_y^2 = (v_A)_y^2 + 2a_c[y_C - y_A]$$

$$0^2 = (13.38 \sin 30°\text{ m/s})^2 + 2(-9.81\text{ m/s}^2)[(h - 1\text{ m}) - 0]$$

$$h = 3.28\text{ m} \qquad \textit{Ans.}$$

NOTE: Show that the bike will strike the ground at B with a velocity having components of

$$(v_B)_x = 11.6\text{ m/s}\rightarrow, \quad (v_B)_y = 8.02\text{ m/s}\downarrow$$

12.7 Curvilinear Motion: Normal and Tangential Components

- Path of motion is describe using *n* and *t* coordinate axes which act normal and tangent to the path
- At the instant considered have their *origin located at the particle*

Planar Motion

- Origin happens to *coincide* with the location of the particle

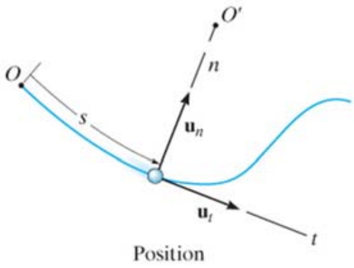


Fig. 12-24 (a)

12.7 Curvilinear Motion: Normal and Tangential Components

Planar Motion

- Curve is constructed from a series of differential arc segments ds
- The plane contains the *n* and *t* axis is referred to as *osculating plane* and is fixed in the plane of motion

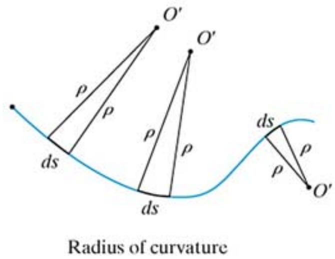


Fig. 12-24 (b)

12.7 Curvilinear Motion: Normal and Tangential Components

Velocity

- Since the particle moves, s is a function of time
- Particle's velocity **v** has a *direction that is always tangent to the path*
- *Magnitude* is determined by taking the time derivative of the path function $s = s(t)$

$$\mathbf{v} = v\mathbf{u}_t \quad (12\text{-}15)$$

where $v = \dot{s}$ (12-16)

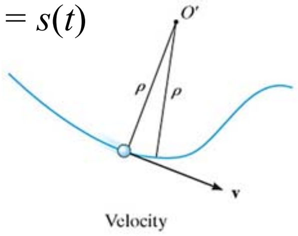
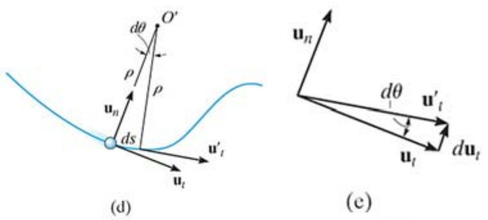


Fig. 12-24

12.7 Curvilinear Motion: No



Acceleration

- Acceleration of the particle is the time rate of change of the velocity

$$\mathbf{a} = \dot{\mathbf{v}} = \dot{v}\mathbf{u}_t + v\dot{\mathbf{u}}_t \quad (12\text{-}17)$$

- **a** can be written as

$$\mathbf{a} = a_t\mathbf{u}_t + a_n\mathbf{u}_n \quad (12\text{-}18)$$

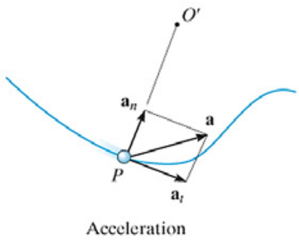


Fig. 12-24

Where $a_t = \dot{v}$ or $a_t ds = v\,dv$ (12-19) and $a_n = \dfrac{v^2}{\rho}$ (12-20)

- Magnitude of acceleration is : $a = \sqrt{a_t^2 + a_n^2}$ (12-21)

12.7 Curvilinear Motion: Normal and Tangential Components

Two special cases of motion

- If the particle moves along a straight line, then $\rho \rightarrow \infty$ and from Eq. 12-20, $a_n = 0$. Thus $a = a_t = \dot{v}$, and we can conclude that the tangential component of acceleration represents the time rate of change in the magnitude of the velocity.
- If the particle moves along a curve with a constant speed, then $a_t = \dot{v}, = 0$ and $a = a_n = v^2 / \rho$. Therefore, the normal component of acceleration represents the time rate of change in the direction of the velocity.

  Since a_n always acts towards the center of curvature, this component is sometimes referred to as the centripetal acceleration.

12.7 Curvilinear Motion: Normal and Tangential Components

- A particle moving along the curved path in Fig. 12-25 will have accelerations directed as shown.

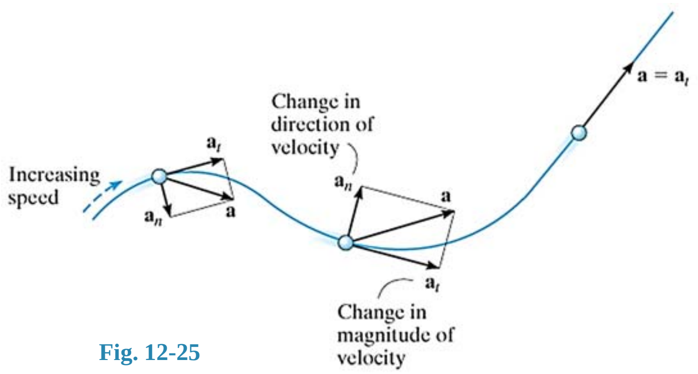


Fig. 12-25

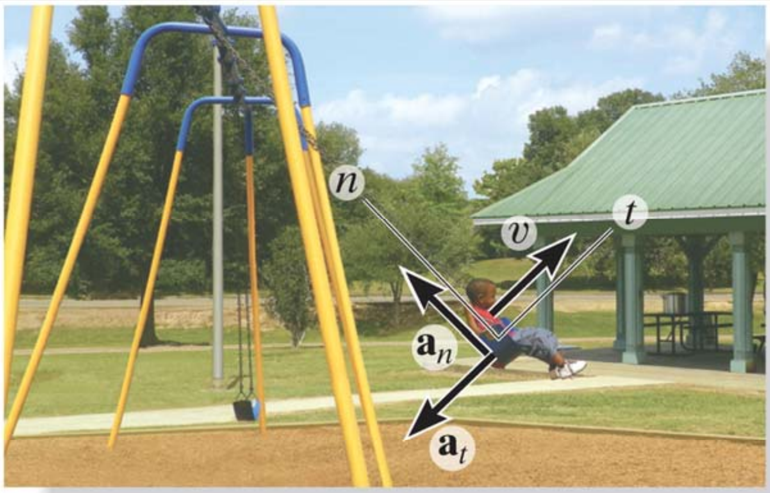


Figure: 12_PH006

As the boy swings upward with a velocity **v**, his motion can be analyzed using n–t coordinates. As he rises, the magnitude of his velocity (speed) is decreasing, and so a_t will be negative. The rate at which the direction of his velocity changes is a_n, which is always positive, that is, towards the center of rotation.

12.7 Curvilinear Motion: Normal and Tangential Components

Three-Dimensional Motion.

- Three unit vectors : $\mathbf{u}_n,\ \mathbf{u}_t,\ \mathbf{u}_b$
- Three unit vectors are related to one another by the vector cross product, e.g. $\mathbf{u}_b = \mathbf{u}_t \times \mathbf{u}_n$

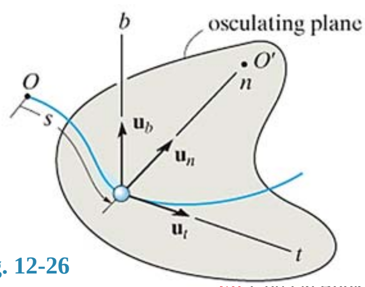


Fig. 12-26

12.7 Curvilinear Motion: Normal and Tangential Components

Procedure for Analysis

Coordinate System.

- Provided the *path* of the particle is *known*, we can establish a set of n and t coordinates having a *fixed origin*, which is coincident with the particle at the instant considered.
- The positive tangent axis acts in the direction of motion and the positive normal axis is directed toward the path's center of curvature.

Velocity.

- The particle's *velocity* is always tangent to the path.
- The magnitude of velocity is found from the time derivative of the path function.

$$v = \dot{s}$$

12.7 Curvilinear Motion: Normal and Tangential Components

Tangential Acceleration.

- The tangential component of acceleration is the result of the time rate of change in the *magnitude* of velocity. This component acts in the positive s direction if the particle's speed is increasing or in the opposite direction if the speed is decreasing.
- The relations between a_t, v, t, and s are the same as for rectilinear motion, namely,

$$a_t = \dot{v} \quad a_t\,ds = v\,dv$$

- If a_t is constant, $a_t = (a_t)_c$, the above equations, when integrated, yield

$$s = s_0 + v_0 t + \tfrac{1}{2}(a_t)_c t^2$$
$$v = v_0 + (a_t)_c t$$
$$v^2 = v_0^2 + 2(a_t)_c(s - s_0)$$

12.7 Curvilinear Motion: Normal and Tangential Components

Normal Acceleration.

- The normal component of acceleration is the result of the time rate of change in the *direction* of the velocity. This component is *always* directed toward the center of curvature of the path, i.e., along the positive n axis.
- The magnitude of this component is determined from

$$a_n = \frac{v^2}{\rho}$$

- If the path is expressed as $y = f(x)$, the radius of curvature ρ at any point on the path is determined from the equation

$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|}$$

The derivation of this result is given in any standard calculus text.

12.7 Curvilinear Motion: Normal and Tangential Components

Once the rotation is constant, the riders will then have only a normal component of acceleration.

12.7 Curvilinear Motion: Normal and Tangential Components

Motorists traveling along this cloverleaf interchange experience a normal acceleration due to the change in direction of their velocity. A tangential component of acceleration occurs when the cars' speed is increased or decreased.

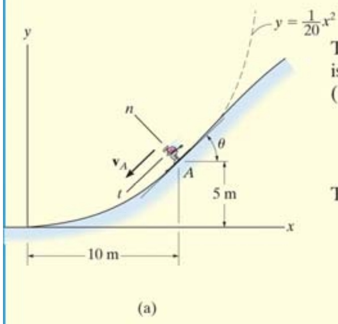
EXAMPLE 12.14

When the skier reaches point A along the parabolic path in Fig. 12–27*a*, he has a speed of 6 m/s which is increasing at 2 m/s^2. Determine the direction of his velocity and the direction and magnitude of his acceleration at this instant. Neglect the size of the skier in the calculation.

SOLUTION

Coordinate System. Although the path has been expressed in terms of its x and y coordinates, we can still establish the origin of the n, t axes at the fixed point A on the path and determine the components of $\mathbf{v}$ and $\mathbf{a}$ along these axes, Fig. 12–27*a*.

EXAMPLE 12.14 CONTINUED



(a)

Velocity. By definition, the velocity is always directed tangent to the path. Since $y = \frac{1}{20}x^2$, $dy/dx = \frac{1}{10}x$, then at $x = 10$ m, $dy/dx = 1$. Hence, at A, $\mathbf{v}$ makes an angle of $\theta = \tan^{-1}1 = 45°$ with the x axis, Fig. 12–27b. Therefore,

$$v_A = 6 \text{ m/s} \quad 45° \nearrow \qquad \textit{Ans.}$$

The acceleration is determined from $\mathbf{a} = \dot{v}\mathbf{u}_t + (v^2/\rho)\mathbf{u}_n$. However, it is first necessary to determine the radius of curvature of the path at A (10 m, 5 m). Since $d^2y/dx^2 = \frac{1}{10}$, then

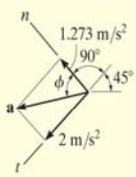
$$\rho = \frac{[1 + (dy/dx)^2]^{3/2}}{|d^2y/dx^2|} = \left.\frac{\left[1 + \left(\frac{1}{10}x\right)^2\right]^{3/2}}{\left|\frac{1}{10}\right|}\right|_{x=10\text{ m}} = 28.28 \text{ m}$$

The acceleration becomes

$$\mathbf{a}_A = \dot{v}\mathbf{u}_t + \frac{v^2}{\rho}\mathbf{u}_n$$
$$= 2\mathbf{u}_t + \frac{(6 \text{ m/s})^2}{28.28 \text{ m}}\mathbf{u}_n$$
$$= \{2\mathbf{u}_t + 1.273\mathbf{u}_n\} \text{ m/s}^2$$

Copyright © 2017 Pearson Education — Gau Lih Book Co., Ltd.

EXAMPLE 12.14 CONTINUED



(b)

Fig. 12–27

As shown in Fig. 12–27b,

$$a = \sqrt{(2 \text{ m/s}^2)^2 + (1.273 \text{ m/s}^2)^2} = 2.37 \text{ m/s}^2$$

$$\phi = \tan^{-1}\frac{2}{1.273} = 57.5°$$

Thus, $45° + 90° + 57.5° - 180° = 12.5°$ so that,

$$a = 2.37 \text{ m/s}^2 \quad 12.5° \nearrow \qquad \textit{Ans.}$$

NOTE: By using n, t coordinates, we were able to readily solve this problem through the use of Eq. 12–18, since it accounts for the separate changes in the magnitude and direction of $\mathbf{v}$.

Copyright © 2017 Pearson Education — Gau Lih Book Co., Ltd.

EXAMPLE 12.15

A race car C travels around the horizontal circular track that has a radius of 300 m, Fig. 12–28. If the car increases its speed at a constant rate of 1.5 m/s^2, starting from rest, determine the time needed for it to reach an acceleration of 2 m/s^2. What is its speed at this instant?

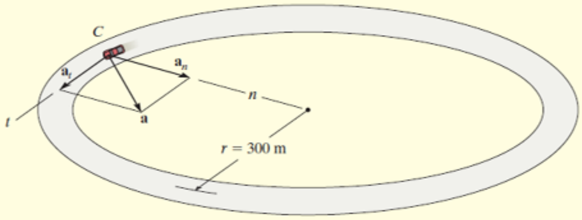


Fig. 12–28

EXAMPLE 12.15 CONTINUED

SOLUTION

Coordinate System. The origin of the n and t axes is coincident with the car at the instant considered. The t axis is in the direction of motion, and the positive n axis is directed toward the center of the circle. This coordinate system is selected since the path is known.

Acceleration. The magnitude of acceleration can be related to its components using $a = \sqrt{a_t^2 + a_n^2}$. Here $a_t = 1.5\ m/s^2$. Since $a_n = v^2/\rho$, the velocity as a function of time must be determined first.

$$v = v_0 + (a_t)_c t$$
$$v = 0 + 1.5t$$

Thus

$$a_n = \frac{v^2}{\rho} = \frac{(1.5t)^2}{300} = 0.0075t^2\ m/s^2$$

The time needed for the acceleration to reach 2 m/s^2 is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$
$$2\ m/s^2 = \sqrt{(1.5\ m/s^2)^2 + (0.0075t^2)^2}$$

Solving for the positive value of t yields

$$0.0075t^2 = \sqrt{(2\ m/s^2)^2 - (1.5\ m/s^2)^2}$$
$$t = 13.28\ s = 13.3\ s \qquad \textit{Ans.}$$

EXAMPLE 12.15 CONTINUED

The time needed for the acceleration to reach 2 m/s² is therefore

$$a = \sqrt{a_t^2 + a_n^2}$$

$$2\text{ m/s}^2 = \sqrt{(1.5\text{ m/s}^2)^2 + (0.0075t^2)^2}$$

Solving for the positive value of t yields

$$0.0075t^2 = \sqrt{(2\text{ m/s}^2)^2 - (1.5\text{ m/s}^2)^2}$$

$$t = 13.28\text{ s} = 13.3\text{ s} \qquad \textit{Ans.}$$

Velocity. The speed at time $t = 13.28$ s is

$$v = 1.5t = 1.5(13.28) = 19.9\text{ m/s} \qquad \textit{Ans.}$$

NOTE: Remember the velocity will always be tangent to the path, whereas the acceleration will be directed within the curvature of the path.

EXAMPLE 12.16

(© R.C. Hibbeler)

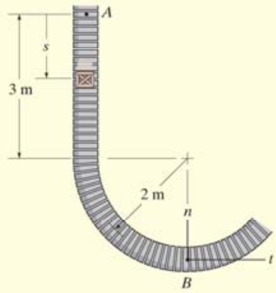
(a)

The boxes in Fig. 12–29a travel along the industrial conveyor. If a box as in Fig. 12–29b starts from rest at A and increases its speed such that $a_t = (0.2t)$ m/s², where t is in seconds, determine the magnitude of its acceleration when it arrives at point B.

SOLUTION

Coordinate System. The position of the box at any instant is defined from the fixed point A using the position or path coordinate s, Fig. 12–29b. The acceleration is to be determined at B, so the origin of the n, t axes is at this point.

EXAMPLE 12.16 CONTINUED



(b)

Acceleration. To determine the acceleration components $a_t = \dot{v}$ and $a_n = v^2/\rho$, it is first necessary to formulate v and $\dot{v}$ so that they may be evaluated at B. Since $v_A = 0$ when $t = 0$, then

$$a_t = \dot{v} = 0.2t \quad (1)$$

$$\int_0^v dv = \int_0^t 0.2t\, dt$$

$$v = 0.1t^2 \quad (2)$$

The time needed for the box to reach point B can be determined by realizing that the position of B is $s_B = 3 + 2\pi(2)/4 = 6.142$ m, Fig. 12–29*b*, and since $s_A = 0$ when $t = 0$ we have

$$v = \frac{ds}{dt} = 0.1t^2$$

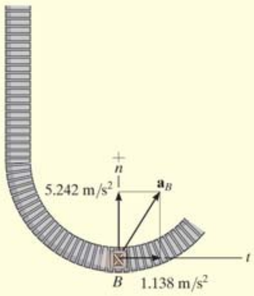
$$\int_0^{6.142\text{ m}} ds = \int_0^{t_B} 0.1t^2 dt$$

$$6.142\text{ m} = 0.0333t_B^3$$

$$t_B = 5.690\text{s}$$

 Gau Lih Book Co., Ltd.

EXAMPLE 12.16 CONTINUED



(c)

Fig. 12–29

Substituting into Eqs. 1 and 2 yields

$$(a_B)_t = \dot{v}_B = 0.2(5.690) = 1.138\text{ m/s}^2$$

$$v_B = 0.1(5.69)^2 = 3.238\text{ m/s}$$

At B, $\rho_B = 2$ m, so that

$$(a_B)_n = \frac{v_B^2}{\rho_B} = \frac{(3.238\text{ m/s})^2}{2\text{ m}} = 5.242\text{ m/s}^2$$

The magnitude of $\mathbf{a}_B$, Fig. 12–29*c*, is therefore

$$a_B = \sqrt{(1.138\text{ m/s}^2)^2 + (5.242\text{ m/s}^2)^2} = 5.36\text{ m/s}^2 \qquad \textit{Ans.}$$

 Gau Lih Book Co., Ltd.

12.8 Curvilinear Motion: Cylindrical Components

Polar Coordinates

- Location of *the particle* use both the *radial coordinate* r and a *traverse coordinate* θ which is counterclockwise angle
- Angle measured in degrees or radians where $1 \text{ rad} = 180^\circ/\pi$

Position

- At any instant, position defined by the position vector

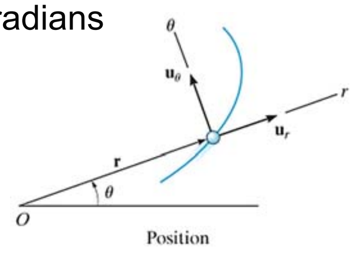


Fig. 12-30 (a)

$$\mathbf{r} = r\mathbf{u}_r \quad \text{(12-22)}$$

12.8 Curvilinear Motion: Cylindrical Components

Velocity

- Instantaneous velocity **v** is obtained by the time derivative of **r**

$$\mathbf{v} = \dot{\mathbf{r}} = \dot{r}\mathbf{u}_r + r\dot{\mathbf{u}}_r$$

- A change $\Delta\theta$ will cause $\mathbf{u}_r$ to become $\mathbf{u}_r$' where $\mathbf{u}_r' = \mathbf{u}_r + \Delta\mathbf{u}_r$
- For small angles $\Delta\theta$, $\Delta\mathbf{u}_r = \Delta\theta\mathbf{u}_\theta$

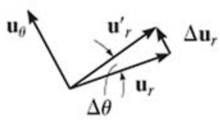


Fig. 12-30 (b)

$$\dot{\mathbf{u}}_r = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u}_r}{\Delta t} = \left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right)\mathbf{u}_\theta$$

$$\dot{\mathbf{u}}_r = \dot{\theta}\mathbf{u}_\theta \quad \text{(12-23)}$$

12.8 Curvilinear Motion: Cylindrical Components

Velocity

- We have $\mathbf{v} = v_r\mathbf{u}_r + v_\theta\mathbf{u}_\theta$ (12-24) where $v_r = \dot{r}$, $v_\theta = r\dot{\theta}$ (12-25)
- Since $\mathbf{v}_r$ *and* $\mathbf{v}_\theta$ are mutually perpendicular,

$$v = \sqrt{(\dot{r})^2 + (r\dot{\theta})^2} \quad (12\text{-}26)$$

- Direction of $\mathbf{v}$ is tangent to the path

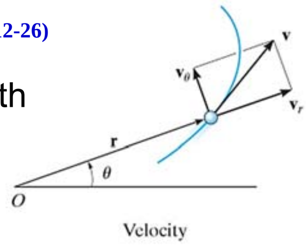


Fig. 12-30 (c)

12.8 Curvilinear Motion: Cylindrical Components

Acceleration

- Taking the time derivatives, we obtain :

$$\mathbf{a} = \dot{\mathbf{v}} = \ddot{r}\mathbf{u}_r + \dot{r}\dot{\mathbf{u}}_r + \dot{r}\dot{\theta}\mathbf{u}_\theta + r\ddot{\theta}\mathbf{u}_\theta + r\dot{\theta}\dot{\mathbf{u}}_\theta$$

- For small angles, $\Delta\mathbf{u}_\theta = -\Delta\theta\mathbf{u}_r$
- Thus,

$$\dot{\mathbf{u}}_\theta = \lim_{\Delta t \to 0} \frac{\Delta \mathbf{u}_\theta}{\Delta t} = -\left(\lim_{\Delta t \to 0} \frac{\Delta\theta}{\Delta t}\right)\mathbf{u}_r$$

$$\dot{\mathbf{u}}_\theta = -\dot{\theta}\mathbf{u}_r \quad (12\text{-}27)$$

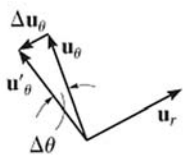


Fig. 12-30 (d)

12.8 Curvilinear Motion: Cylindrical Components

Acceleration

- We can write the acceleration in component form as

$$\mathbf{a} = a_r\mathbf{u}_r + a_\theta\mathbf{u}_\theta \quad \text{(12-28)}$$

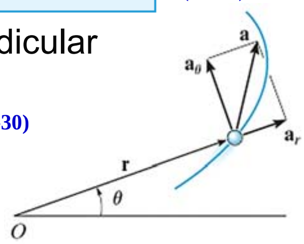
where

$$a_r = \ddot{r} - r\dot{\theta}^2$$
$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} \quad \text{(12-29)}$$

- Since $\mathbf{a}_r$ and $\mathbf{a}_\theta$ are always perpendicular

$$a = \sqrt{(\ddot{r} - r\dot{\theta}^2)^2 + (r\ddot{\theta} + 2\dot{r}\dot{\theta})^2} \quad \text{(12-30)}$$

- Acceleration will not be tangent to the path



Acceleration

Fig. 12-30 (c)

12.8 Curvilinear Motion: Cylindrical Components

Cylindrical Coordinates

- When the particle moves along a space, location is specified by the three cylindrical coordinates r, θ, z
- Position, velocity, acceleration of the particle is written as

$$\mathbf{r}_P = r\mathbf{u}_r + z\mathbf{u}_z$$
$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta + \dot{z}\mathbf{u}_z \quad \text{(12-31)}$$
$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta + \ddot{z}\mathbf{u}_z \quad \text{(12-32)}$$

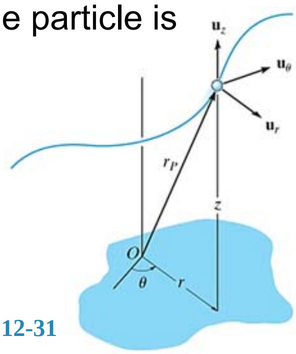


Fig. 12-31

12.8 Curvilinear Motion: Cylindrical Components

Time Derivatives

2 common problems:

1. If the polar coordinates are specified as $r = r(t)$ and $\theta = \theta(t)$, time derivatives can be found directly.
2. If the time-parametric equations are not given, the path $r = f(\theta)$ must be known and using the chain rule of calculus can find the relation between the time derivatives.

12.8 ... ts

Procedure for Analysis

Coordinate System.

- Polar coordinates are a suitable choice for solving problems when data regarding the angular motion of the radial coordinate r is given to describe the particle's motion. Also, some paths of motion can conveniently be described in terms of these coordinates.
- To use polar coordinates, the origin is established at a fixed point, and the radial line r is directed to the particle.
- The transverse coordinate θ is measured from a fixed reference line to the radial line.

Velocity and Acceleration.

- Once r and the four time derivatives $\dot{r}$, $\ddot{r}$, $\dot{\theta}$, and $\ddot{\theta}$ have been evaluated at the instant considered, their values can be substituted into Eqs. 12–25 and 12–29 to obtain the radial and transverse components of **v** and **a**.
- If it is necessary to take the time derivatives of $r = f(\theta)$, then the chain rule of calculus must be used. See Appendix C.
- Motion in three dimensions requires a simple extension of the above procedure to include $\dot{z}$ and $\ddot{z}$.

12.8 Curvilinear Motion: Cylindrical Components

The spiral motion of this girl can be followed by using cylindrical components. Here the radial coordinate r is constant, the transverse coordinate θ will increase with time as the girl rotates about the vertical, and her altitude z will decrease with time.

EXAMPLE 12.17

The amusement park ride shown in Fig. 12–32*a* consists of a chair that is rotating in a horizontal circular path of radius r such that the arm OB has an angular velocity $\dot{\theta}$ and angular acceleration $\ddot{\theta}$. Determine the radial and transverse components of velocity and acceleration of the passenger. Neglect his size in the calculation.

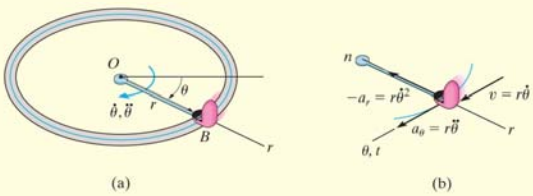


Fig. 12–32

SOLUTION

Coordinate System. Since the angular motion of the arm is reported, polar coordinates are chosen for the solution, Fig. 12–32*a*. Here θ is not related to r, since the radius is constant for all θ.

EXAMPLE 12.17 CONTINUED

Velocity and Acceleration. It is first necessary to specify the first and second time derivatives of r and θ. Since r is *constant*, we have

$$r = r \qquad \dot{r} = 0 \qquad \ddot{r} = 0$$

Thus,

$$v_r = \dot{r} = 0 \qquad \textit{Ans.}$$

$$v_\theta = r\dot{\theta} \qquad \textit{Ans.}$$

$$a_r = \ddot{r} - r\dot{\theta}^2 = -r\dot{\theta}^2 \qquad \textit{Ans.}$$

$$a_\theta = r\ddot{\theta} + 2\dot{r}\dot{\theta} = r\ddot{\theta} \qquad \textit{Ans.}$$

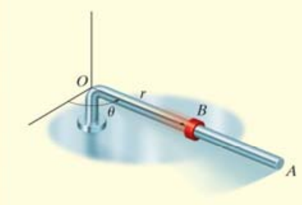
These results are shown in Fig. 12–32*b*.

NOTE: The n, t axes are also shown in Fig. 12–32*b*, which in this special case of circular motion happen to be *collinear* with the r and θ axes, respectively. Since $v = v_\theta = v_t = r\dot{\theta}$, then by comparison,

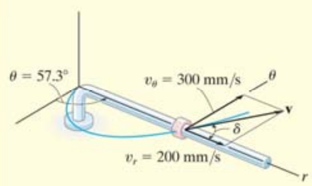
$$-a_r = a_n = \frac{v^2}{\rho} = \frac{(r\dot{\theta})^2}{r} = r\dot{\theta}^2$$

$$a_\theta = a_t = \frac{dv}{dt} = \frac{d}{dt}(r\dot{\theta}) = \frac{dr}{dt}\dot{\theta} + r\frac{d\dot{\theta}}{dt} = 0 + r\ddot{\theta}$$

EXAMPLE 12.18



(a)



(b)

The rod OA in Fig. 12–33*a* rotates in the horizontal plane such that $\theta = (t^3)$ rad. At the same time, the collar B is sliding outward along OA so that $r = (100t^2)$ mm. If in both cases t is in seconds, determine the velocity and acceleration of the collar when $t = 1$ s.

SOLUTION

Coordinate System. Since time-parametric equations of the path are given, it is not necessary to relate r to θ.

Velocity and Acceleration. Determining the time derivatives and evaluating them when $t = 1$ s, we have

$$r = 100t^2\Big|_{t=1\text{ s}} = 100\text{ mm} \qquad \theta = t^3\Big|_{t=1\text{ s}} = 1\text{ rad} = 57.3^\circ$$

$$\dot{r} = 200t\Big|_{t=1\text{ s}} = 200\text{ mm/s} \qquad \dot{\theta} = 3t^2\Big|_{t=1\text{ s}} = 3\text{ rad/s}$$

$$\ddot{r} = 200\Big|_{t=1\text{ s}} = 200\text{ mm/s}^2 \qquad \ddot{\theta} = 6t\Big|_{t=1\text{ s}} = 6\text{ rad/s}^2.$$

As shown in Fig. 12–33*b*,

$$\mathbf{v} = \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta$$

$$= 200\mathbf{u}_r + 100(3)\mathbf{u}_\theta = \{200\mathbf{u}_r + 300\mathbf{u}_\theta\}\text{ mm/s}$$

EXAMPLE 12.18 CONTINUED

The magnitude of **v** is

$$v = \sqrt{(200)^2 + (300)^2} = 361 \text{ mm/s} \qquad \textit{Ans.}$$

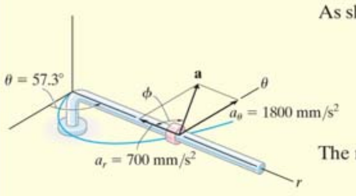
$$\delta = \tan^{-1}\left(\frac{300}{200}\right) = 56.3° \quad \delta + 57.3° = 114° \qquad \textit{Ans.}$$

As shown in Fig. 12–33*c*,

$$\mathbf{a} = (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta$$

$$= [200 - 100(3)^2]\mathbf{u}_r + [100(6) + 2(200)3]\mathbf{u}_\theta$$

$$= \{-700\mathbf{u}_r + 1800\mathbf{u}_\theta\} \text{ mm/s}^2$$



(c)

Fig. 12–33

The magnitude of **a** is

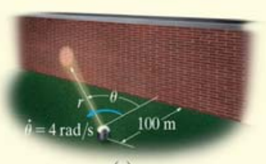
$$a = \sqrt{(-700)^2 + (1800)^2} = 1930 \text{ mm/s}^2 \qquad \textit{Ans.}$$

$$\phi = \tan^{-1}\left(\frac{1800}{700}\right) = 68.7° \quad (180° - \phi) + 57.3° = 169° \qquad \textit{Ans.}$$

NOTE: The velocity is tangent to the path; however, the acceleration is directed within the curvature of the path, as expected.

EXAMPLE 12.19

The searchlight in Fig. 12–34*a* casts a spot of light along the face of a wall that is located 100 m from the searchlight. Determine the magnitudes of the velocity and acceleration at which the spot appears to travel across the wall at the instant $\theta = 45°$. The searchlight rotates at a constant rate of $\dot{\theta} = 4$ rad/s.



(a)

SOLUTION

Coordinate System. Polar coordinates will be used to solve this problem since the angular rate of the searchlight is given. To find the necessary time derivatives it is first necessary to relate *r* to *θ*. From Fig. 12–34*a*,

$$r = 100/\cos\theta = 100 \sec\theta$$

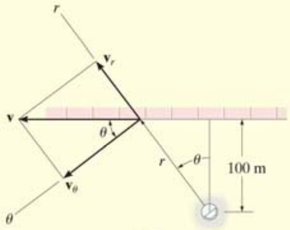
EXAMPLE 12.19 CONTINUED

Velocity and Acceleration. Using the chain rule of calculus, noting that $d(\sec\theta) = \sec\theta\tan\theta\, d\theta$, and $d(\tan\theta) = \sec^2\theta\, d\theta$, we have

$$\begin{aligned}\dot{r} &= 100(\sec\theta\tan\theta)\dot{\theta}\\ \ddot{r} &= 100(\sec\theta\tan\theta)\dot{\theta}(\tan\theta)\dot{\theta} + 100\sec\theta(\sec^2\theta)\dot{\theta}(\dot{\theta})\\ &\quad + 100\sec\theta\tan\theta(\ddot{\theta})\\ &= 100\sec\theta\tan^2\theta(\dot{\theta})^2 + 100\sec^3\theta(\dot{\theta})^2 + 100(\sec\theta\tan\theta)\ddot{\theta}\end{aligned}$$

Since $\dot{\theta} = 4$ rad/s = constant, then $\ddot{\theta} = 0$, and the above equations, when $\theta = 45°$, become

$$\begin{aligned}r &= 100\sec 45° = 141.4\\ \dot{r} &= 400\sec 45°\tan 45° = 565.7\\ \ddot{r} &= 1600(\sec 45°\tan^2 45° + \sec^3 45°) = 6788.2\end{aligned}$$



(b)

EXAMPLE 12.19 CONTINUED

As shown in Fig. 12–34*b*,

$$\begin{aligned}\mathbf{v} &= \dot{r}\mathbf{u}_r + r\dot{\theta}\mathbf{u}_\theta\\ &= 565.7\mathbf{u}_r + 141.4(4)\mathbf{u}_\theta\\ &= \{565.7\mathbf{u}_r + 565.7\mathbf{u}_\theta\}\text{ m/s}\\ v &= \sqrt{v_r^2 + v_\theta^2} = \sqrt{(565.7)^2 + (565.7)^2}\\ &= 800\text{ m/s}\end{aligned}$$ *Ans.*

As shown in Fig. 12–34c,

$$\begin{aligned}\mathbf{a} &= (\ddot{r} - r\dot{\theta}^2)\mathbf{u}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\mathbf{u}_\theta\\ &= [6788.2 - 141.4(4)^2]\mathbf{u}_r + [141.4(0) + 2(565.7)4]\mathbf{u}_\theta\\ &= \{4525.5\mathbf{u}_r + 4525.5\mathbf{u}_\theta\}\text{ m/s}^2\\ a &= \sqrt{a_r^2 + a_\theta^2} = \sqrt{(4525.5)^2 + (4525.5)^2}\\ &= 6400\text{ m/s}^2\end{aligned}$$ *Ans.*

NOTE: It is also possible to find a without having to calculate $\ddot{r}$ (or a_r). As shown in Fig. 12–34*d*, since $a_\theta = 4525.5$ m/s², then by vector resolution, $a = 4525.5/\cos 45° = 6400$ m/s².

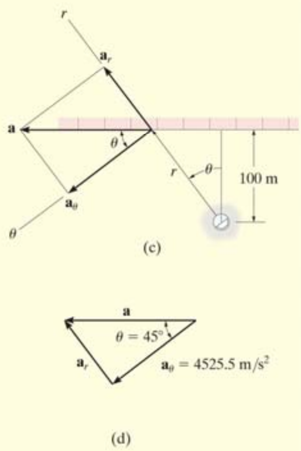
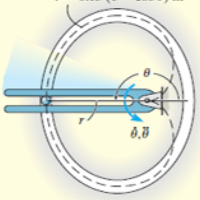


Fig. 12–34

EXAMPLE 12.20



(a)

Due to the rotation of the forked rod, the ball in Fig. 12–35*a* travels around the slotted path, a portion of which is in the shape of a cardioid, $r = 0.15(1 - \cos\theta)$ m, where θ is in radians. If the ball's velocity is $v = 1.2$ m/s and its acceleration is $a = 9\ \text{m/s}^2$ at the instant $\theta = 180°$, determine the angular velocity $\dot\theta$ and angular acceleration $\ddot\theta$ of the fork.

SOLUTION

Coordinate System. This path is most unusual, and mathematically it is best expressed using polar coordinates, as done here, rather than rectangular coordinates. Also, since $\dot\theta$ and $\ddot\theta$ must be determined, then r, θ coordinates are an obvious choice.

Velocity and Acceleration. The time derivatives of r and θ can be determined using the chain rule.

$$r = 0.15(1 - \cos\theta)$$
$$\dot r = 0.15(\sin\theta)\dot\theta$$
$$\ddot r = 0.15(\cos\theta)\dot\theta(\dot\theta) + 0.15(\sin\theta)\ddot\theta$$

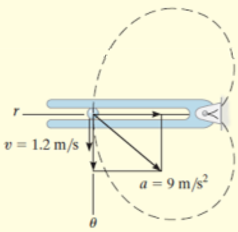
Evaluating these results at $\theta = 180°$, we have

$$r = 0.3\ \text{m} \qquad \dot r = 0 \qquad \ddot r = -0.15\dot\theta^2$$

Since $v = 1.2$ m/s, using Eq. 12–26 to determine $\dot\theta$ yields

$$v = \sqrt{(\dot r)^2 + (r\dot\theta)^2}$$
$$1.2 = \sqrt{(0)^2 + (0.3\dot\theta)^2}$$
$$\dot\theta = 4\ \text{rad/s} \qquad \textit{Ans.}$$

EXAMPLE 12.20 CONTINUED



(b)

Fig. 12–35

In a similar manner, $\ddot\theta$ can be found using Eq. 12–30.

$$a = \sqrt{(\ddot r - r\dot\theta^2)^2 + (r\ddot\theta + 2\dot r\dot\theta)^2}$$
$$9 = \sqrt{[-0.15(4)^2 - 0.3(4)^2]^2 + [0.3\ddot\theta + 2(0)(4)]^2}$$
$$(9)^2 = (-7.2)^2 + 0.09\,\ddot\theta^2$$
$$\ddot\theta = 18\ \text{rad/s}^2 \qquad \textit{Ans.}$$

Vectors **a** and **v** are shown in Fig. 12–35*b*.

NOTE: At this location, the θ and t (tangential) axes will coincide. The $+n$ (normal) axis is directed to the right, opposite to $+r$.

12.9 Absolute Dependent Motion Analysis of Two Particles

- Motion of one particle *depend* on the corresponding motion of another particle
- Movement of *A* downward along the inclined plane will cause a movement of *B* up the other incline
- If the total cord length is l_T, the two position coordinates are related by the equation

$$s_A + l_{CD} + s_B = l_T$$

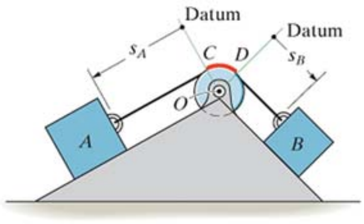


Fig. 12-36

- The negative sign indicates *A* has a velocity downward

$$\frac{ds_A}{dt} + \frac{ds_B}{dt} = 0 \qquad \text{or} \qquad v_B = -v_A$$

12.9 Absolute Dependent Motion Analysis of Two Particles

- Time differentiation of the velocities yields the relation between accelerations : $a_B = - a_A$
- *A* is specified by s_A, and the position of the end of the cord from which block *B* is suspended is defined by s_B
- Position coordinate can be related by

$$2s_B + h + s_A = l$$

- Since l and h are constant during the motion,

$$2v_B = -v_A \quad , \quad 2a_B = -a_A$$

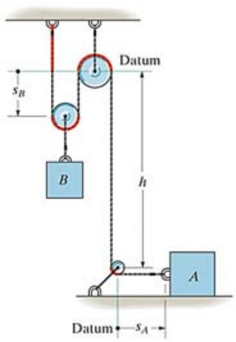


Fig. 12-37 (a)

12.9 Absolute Dependent Motion Analysis of Two Particles

- Defining the position of block *B* from the center of the bottom pulley (a fixed point),

$$2(h-s_B)+h+s_A=l$$

- Time differentiation yields

$$2v_B=v_A \qquad 2a_B=a_A$$

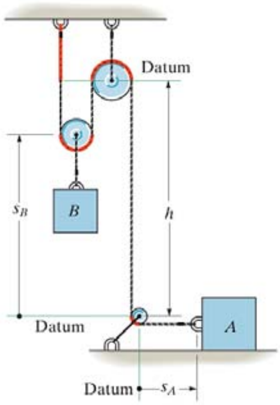


Fig. 12-37

12.9 ... articles

Procedure for Analysis

The above method of relating the dependent motion of one particle to that of another can be performed using algebraic scalars or position coordinates provided each particle moves along a rectilinear path. When this is the case, only the magnitudes of the velocity and acceleration of the particles will change, not their line of direction.

Position-Coordinate Equation.

- Establish each position coordinate with an origin located at a *fixed* point or datum.
- It is *not necessary* that the *origin* be the *same* for each of the coordinates; however, it is *important* that each coordinate axis selected be directed along the *path of motion* of the particle.
- Using geometry or trigonometry, relate the position coordinates to the total length of the cord, l_T, or to that portion of cord, l, which *excludes* the segments that do not change length as the particles move—such as arc segments wrapped over pulleys.
- If a problem involves a *system* of two or more cords wrapped around pulleys, then the position of a point on one cord must be related to the position of a point on another cord using the above procedure. Separate equations are written for a fixed length of each cord of the system and the positions of the two particles are then related by these equations (see Examples 12.22 and 12.23).

12.9 Absolute Dependent Motion Analysis of Two Particles

Time Derivatives.

- Two successive time derivatives of the position-coordinate equations yield the required velocity and acceleration equations which relate the motions of the particles.
- The signs of the terms in these equations will be consistent with those that specify the positive and negative sense of the position coordinates.

12.9 Absolute Dependent Motion Analysis of Two Particles

The cable is wrapped around the pulleys on this crane in order to reduce the required force needed to hoist a load.

EXAMPLE 12.21

Determine the speed of block A in Fig. 12–38 if block B has an upward speed of 6 m/s.

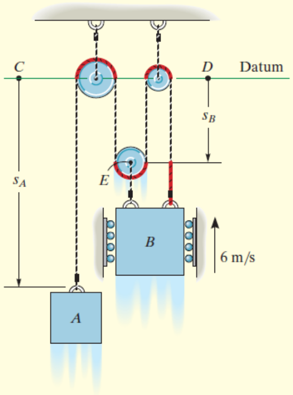


Fig. 12–38

EXAMPLE 12.21 CONTINUED

SOLUTION

Position-Coordinate Equation. There is *one cord* in this system having segments which change length. Position coordinates s_A and s_B will be used since each is measured from a fixed point (C or D) and extends along each block's *path of motion*. In particular, s_B is directed to point E since motion of B and E is the *same*.

The red colored segments of the cord in Fig. 12–38 remain at a constant length and do not have to be considered as the blocks move. The remaining length of cord, l, is also constant and is related to the changing position coordinates s_A and s_B by the equation

$$s_A + 3s_B = l$$

Time Derivative. Taking the time derivative yields

$$v_A + 3v_B = 0$$

so that when $v_B = -6$ m/s (upward),

$$v_A = 18 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.22

Determine the speed of A in Fig. 12–39 if B has an upward speed of 6 m/s.

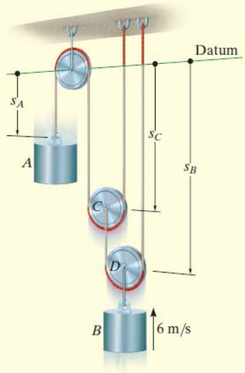


Fig. 12–39

EXAMPLE 12.22 CONTINUED

SOLUTION

Position-Coordinate Equation. As shown, the positions of blocks A and B are defined using coordinates s_A and s_B. Since the system has *two cords* with segments that change length, it will be necessary to use a third coordinate, s_C, in order to relate s_A to s_B. In other words, the length of one of the cords can be expressed in terms of s_A and s_C, and the length of the other cord can be expressed in terms of s_B and s_C.

The red colored segments of the cords in Fig. 12–39 do not have to be considered in the analysis. Why? For the remaining cord lengths, say l_1 and l_2, we have

$$s_A + 2s_C = l_1 \qquad s_B + (s_B - s_C) = l_2$$

Time Derivative. Taking the time derivative of these equations yields

$$v_A + 2v_C = 0 \qquad 2v_B - v_C = 0$$

Eliminating v_C produces the relationship between the motions of each cylinder.

$$v_A + 4v_B = 0$$

so that when $v_B = -6$ m/s (upward),

$$v_A = +24 \text{ m/s} = 24 \text{ m/s} \downarrow \qquad \textit{Ans.}$$

EXAMPLE 12.23

Determine the speed of block B in Fig. 12–40 if the end of the cord at A is pulled down with a speed of 2 m/s.

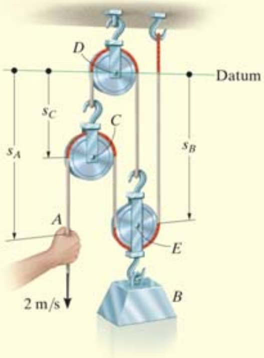


Fig. 12–40

EXAMPLE 12.23 CONTINUED

SOLUTION

Position-Coordinate Equation. The position of point A is defined by s_A, and the position of block B is specified by s_B since point E on the pulley will have the *same motion* as the block. Both coordinates are measured from a horizontal datum passing through the *fixed* pin at pulley D. Since the system consists of *two* cords, the coordinates s_A and s_B cannot be related directly. Instead, by establishing a third position coordinate, s_C, we can now express the length of one of the cords in terms of s_B and s_C, and the length of the other cord in terms of s_A, s_B, and s_C.

Excluding the red colored segments of the cords in Fig. 12–40, the remaining constant cord lengths l_1 and l_2 (along with the hook and link dimensions) can be expressed as

$$s_C + s_B = l_1$$
$$(s_A - s_C) + (s_B - s_C) + s_B = l_2$$

Time Derivative. The time derivative of each equation gives

$$v_C + v_B = 0$$
$$v_A - 2v_C + 2v_B = 0$$

Eliminating v_C, we obtain

$$v_A + 4v_B = 0$$

so that when $v_A = 2$ m/s (downward),

$$v_B = -0.5 \text{ m/s} = 0.5 \text{ m/s} \uparrow \qquad \textit{Ans.}$$

Copyright © 2017 Pearson Education Gau Lih Book Co., Ltd.

EXAMPLE 12.24

A man at A is hoisting a safe S as shown in Fig. 12–41 by walking to the right with a constant velocity $v_A = 0.5\ \text{m/s}$. Determine the velocity and acceleration of the safe when it reaches the elevation of 10 m. The rope is 30 m long and passes over a small pulley at D.

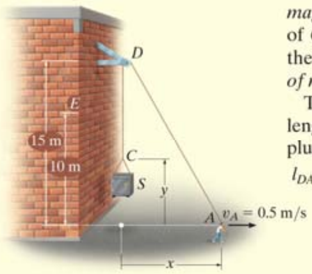


Fig. 12–41

SOLUTION

Position-Coordinate Equation. This problem is unlike the previous examples since rope segment DA changes *both direction and magnitude*. However, the ends of the rope, which define the positions of C and A, are specified by means of the x and y coordinates since they must be measured from a fixed point and *directed along the paths of motion* of the ends of the rope.

The x and y coordinates may be related since the rope has a fixed length $l = 30$ m, which at all times is equal to the length of segment DA plus CD. Using the Pythagorean theorem to determine l_{DA}, we have $l_{DA} = \sqrt{(15)^2 + x^2}$; also, $l_{CD} = 15 - y$. Hence,

$$l = l_{DA} + l_{CD}$$

$$30 = \sqrt{(15)^2 + x^2} + (15 - y)$$

$$y = \sqrt{225 + x^2} - 15 \quad (1)$$

EXAMPLE 12.24 CONTINUED

Time Derivatives. Taking the time derivative, using the chain rule (see Appendix C), where $v_S = dy/dt$ and $v_A = dx/dt$, yields

$$v_S = \frac{dy}{dt} = \left[\frac{1}{2}\frac{2x}{\sqrt{225 + x^2}}\right]\frac{dx}{dt}$$

$$= \frac{x}{\sqrt{225 + x^2}}v_A \quad (2)$$

At $y = 10$ m, x is determined from Eq. 1, i.e., $x = 20$ m. Hence, from Eq. 2 with $v_A = 0.5\ \text{m/s}$,

$$v_S = \frac{20}{\sqrt{225 + (20)^2}}(0.5) = 0.4\ \text{m/s} = 400\ \text{mm/s}\uparrow \qquad \textit{Ans.}$$

The acceleration is determined by taking the time derivative of Eq. 2. Since v_A is constant, then $a_A = dv_A/dt = 0$, and we have

$$a_S = \frac{d^2y}{dt^2} = \left[\frac{-x(dx/dt)}{(225 + x^2)^{3/2}}\right]xv_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]\left(\frac{dx}{dt}\right)v_A + \left[\frac{1}{\sqrt{225 + x^2}}\right]x\frac{dv_A}{dt} = \frac{225v_A^2}{(225 + x^2)^{3/2}}$$

At $x = 20$ m, with $v_A = 0.5\ \text{m/s}$, the acceleration becomes

$$a_S = \frac{225(0.5\ \text{m/s})^2}{[225 + (20\ \text{m})^2]^{3/2}} = 0.00360\ \text{m/s}^2 = 3.60\ \text{mm/s}^2\uparrow \qquad \textit{Ans.}$$

NOTE: The constant velocity at A causes the other end C of the rope to have an acceleration since $\mathbf{v}_A$ causes segment DA to change its direction as well as its length.

12.10 Relative Motion Analysis of Two Particles Using Translating Axes

- There are cases where the path of motion for a particle is complicated
- It may be easier to analyze the motion in parts by using two or more frames of reference

Position

- *Absolute position* of $\mathbf{r}_A$ and $\mathbf{r}_B$ is measured from O of the fixed x, y, z reference frame

$$\mathbf{r}_B = \mathbf{r}_A + \mathbf{r}_{B/A} \quad \text{(12-33)}$$

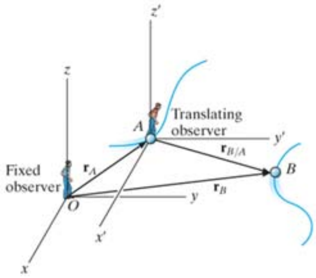


Fig. 12-42

12.10 Relative Motion Analysis of Two Particles Using Translating Axes

Velocity

- By taking the time derivatives, $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$ (12-34)
- $\mathbf{v}_B = d\mathbf{r}_B / dt$ and $\mathbf{v}_A = d\mathbf{r}_A / dt$ refer to *absolute velocities*
- Relative velocity $\mathbf{v}_{B/A} = d\mathbf{r}_{B/A} / dt$ is observed from the translating frame

Acceleration

- The time derivative yields : $\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$ (12-35)

12.10 Relative Motion Analysis of Two Particles Using Translating Axes

Procedure for Analysis

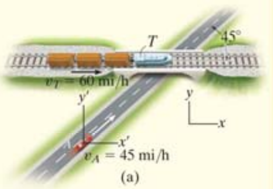
- When applying the relative velocity and acceleration equations, it is first necessary to specify the particle A that is the origin for the translating x', y', z' axes. Usually this point has a *known* velocity or acceleration.
- Since vector addition forms a triangle, there can be at most *two unknowns*, represented by the magnitudes and/or directions of the vector quantities.
- These unknowns can be solved for either graphically, using trigonometry (law of sines, law of cosines), or by resolving each of the three vectors into rectangular or Cartesian components, thereby generating a set of scalar equations.

12.10 Relative Motion Analysis of Two Particles Using Translating Axes

The pilots of these close-flying planes must be aware of their relative positions and velocities at all times in order to avoid a collision.

EXAMPLE 12.25

A train travels at a constant speed of 60 mi/h and crosses over a road as shown in Fig. 12–43*a*. If the automobile *A* is traveling at 45 mi/h along the road, determine the magnitude and direction of the velocity of the train relative to the automobile.



(a)

SOLUTION I

Vector Analysis. The relative velocity $\mathbf{v}_{T/A}$ is measured from the translating x', y' axes attached to the automobile, Fig. 12–43*a*. It is determined from $\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$. Since $\mathbf{v}_T$ and $\mathbf{v}_A$ are known in *both* magnitude and direction, the unknowns become the *x* and *y* components of $\mathbf{v}_{T/A}$. Using the *x*, *y* axes in Fig. 12–43*a*, we have

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$60\mathbf{i} = (45 \cos 45° \mathbf{i} + 45 \sin 45° \mathbf{j}) + \mathbf{v}_{T/A}$$

$$\mathbf{v}_{T/A} = \{28.2\mathbf{i} - 31.8\mathbf{j}\} \text{ mi/h}$$

EXAMPLE 12.25 CONTINUED

The magnitude of $\mathbf{v}_{T/A}$ is thus

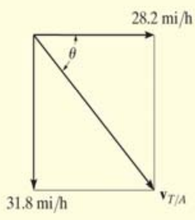
$$v_{T/A} = \sqrt{(28.2)^2 + (-31.8)^2} = 42.5 \text{ mi/h} \qquad \textit{Ans.}$$

From the direction of each component, Fig. 12–43*b*, the direction of $\mathbf{v}_{T/A}$ is

$$\tan \theta = \frac{(v_{T/A})_y}{(v_{T/A})_x} = \frac{31.8}{28.2}$$

$$\theta = 48.5° \qquad \textit{Ans.}$$

Note that the vector addition shown in Fig. 12–43*b* indicates the correct sense for $\mathbf{v}_{T/A}$. This figure anticipates the answer and can be used to check it.



(b)

EXAMPLE 12.25 CONTINUED

SOLUTION II

Scalar Analysis. The unknown components of $\mathbf{v}_{T/A}$ can also be determined by applying a scalar analysis. We will assume these components act in the *positive* x and y directions. Thus,

$$\mathbf{v}_T = \mathbf{v}_A + \mathbf{v}_{T/A}$$

$$\begin{bmatrix} 60 \text{ mi/h} \\ \rightarrow \end{bmatrix} = \begin{bmatrix} 45 \text{ mi/h} \\ \measuredangle^{45^\circ} \end{bmatrix} + \begin{bmatrix} (v_{T/A})_x \\ \rightarrow \end{bmatrix} + \begin{bmatrix} (v_{T/A})_y \\ \uparrow \end{bmatrix}$$

Resolving each vector into its x and y components yields

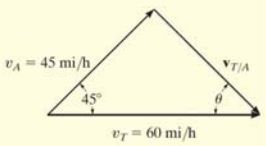
$$(\overset{+}{\rightarrow}) \qquad 60 = 45 \cos 45^\circ + (v_{T/A})_x + 0$$

$$(+\uparrow) \qquad 0 = 45 \sin 45^\circ + 0 + (v_{T/A})_y$$

Solving, we obtain the previous results,

$$(v_{T/A})_x = 28.2 \text{ mi/h} = 28.2 \text{ mi/h} \rightarrow$$

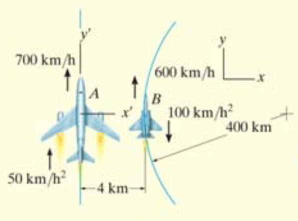
$$(v_{T/A})_y = -31.8 \text{ mi/h} = 31.8 \text{ mi/h} \downarrow$$



(c)

Fig. 12–43

EXAMPLE 12.26



(a)

Plane A in Fig. 12–44a is flying along a straight-line path, whereas plane B is flying along a circular path having a radius of curvature of $\rho_B = 400$ km. Determine the velocity and acceleration of B as measured by the pilot of A.

SOLUTION

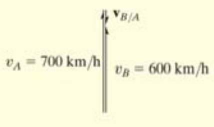
Velocity. The origin of the x *and* y axes are located at an arbitrary fixed point. Since the motion relative to plane A is to be determined, the *translating frame of reference* x', y' is attached to it, Fig. 12–44a. Applying the relative-velocity equation in scalar form since the velocity vectors of both planes are parallel at the instant shown, we have

$$(+\uparrow) \qquad v_B = v_A + v_{B/A}$$

$$600 \text{ km/h} = 700 \text{ km/h} + v_{B/A}$$

$$v_{B/A} = -100 \text{ km/h} = 100 \text{ km/h} \downarrow \qquad \textit{Ans.}$$

The vector addition is shown in Fig. 12–44b.



(b)

EXAMPLE 12.26 CONTINUED

Acceleration. Plane B has both tangential and normal components of acceleration since it is flying along a *curved path*. From Eq. 12–20, the magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(600 \text{ km/h})^2}{400 \text{ km}} = 900 \text{ km/h}^2$$

Applying the relative-acceleration equation gives

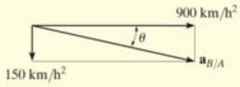
$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$900\mathbf{i} - 100\mathbf{j} = 50\mathbf{j} + \mathbf{a}_{B/A}$$

Thus,

$$\mathbf{a}_{B/A} = \{900\mathbf{i} - 150\mathbf{j}\} \text{ km/h}^2$$

From Fig. 12–44c, the magnitude and direction of $\mathbf{a}_{B/A}$ are therefore

$$a_{B/A} = 912 \text{ km/h}^2 \quad \theta = \tan^{-1}\frac{150}{900} = 9.46° \quad \textit{Ans.}$$



(c)

Fig. 12–44

NOTE: The solution to this problem was possible using a translating frame of reference, since the pilot in plane A is "translating." Observation of the motion of plane A with respect to the pilot of plane B, however, must be obtained using a *rotating* set of axes attached to plane B. (This assumes, of course, that the pilot of B is fixed in the rotating frame, so he does not turn his eyes to follow the motion of A.) The analysis for this case is given in Example 16.21.

EXAMPLE 12.27

At the instant shown in Fig. 12–45*a*, cars A and B are traveling with speeds of 18 m/s and 12 m/s, respectively. Also at this instant, A has a decrease in speed of 2 m/s², and B has an increase in speed of 3 m/s². Determine the velocity and acceleration of B with respect to A.

SOLUTION

Velocity. The fixed x, y axes are established at an arbitrary point on the ground and the translating x', y' axes are attached to car A, Fig. 12–45*a*. Why? The relative velocity is determined from $\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$. What are the two unknowns? Using a Cartesian vector analysis, we have

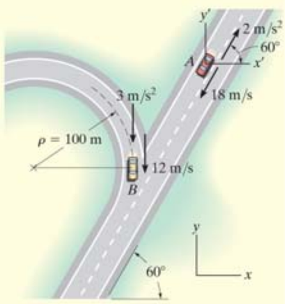
$$\mathbf{v}_B = \mathbf{v}_A + \mathbf{v}_{B/A}$$
$$-12\mathbf{j} = (-18\cos 60°\mathbf{i} - 18\sin 60°\mathbf{j}) + \mathbf{v}_{B/A}$$
$$\mathbf{v}_{B/A} = \{9\mathbf{i} + 3.588\mathbf{j}\} \text{ m/s}$$

Thus,

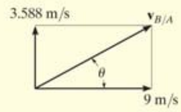
$$v_{B/A} = \sqrt{(9)^2 + (3.588)^2} = 9.69 \text{ m/s} \quad \textit{Ans.}$$

Noting that $\mathbf{v}_{B/A}$ has $+\mathbf{i}$ and $+\mathbf{j}$ components, Fig. 12–45*b*, its direction is

$$\tan\theta = \frac{(v_{B/A})_y}{(v_{B/A})_x} = \frac{3.588}{9}$$
$$\theta = 21.7° \quad \textit{Ans.}$$



(a)



(b)

EXAMPLE 12.27 CONTINUED

Acceleration. Car B has both tangential and normal components of acceleration. Why? The magnitude of the normal component is

$$(a_B)_n = \frac{v_B^2}{\rho} = \frac{(12\ \text{m/s})^2}{100\ \text{m}} = 1.440\ \text{m/s}^2$$

Applying the equation for relative acceleration yields

$$\mathbf{a}_B = \mathbf{a}_A + \mathbf{a}_{B/A}$$
$$(-1.440\mathbf{i} - 3\mathbf{j}) = (2\cos 60°\mathbf{i} + 2\sin 60°\mathbf{j}) + \mathbf{a}_{B/A}$$
$$\mathbf{a}_{B/A} = \{-2.440\mathbf{i} - 4.732\mathbf{j}\}\ \text{m/s}^2$$

Here $\mathbf{a}_{B/A}$ has $-\mathbf{i}$ and $-\mathbf{j}$ components. Thus, from Fig. 12–45c,

$$a_{B/A} = \sqrt{(2.440)^2 + (4.732)^2} = 5.32\ \text{m/s}^2 \qquad \textit{Ans.}$$

$$\tan\phi = \frac{(a_{B/A})_y}{(a_{B/A})_x} = \frac{4.732}{2.440}$$

$$\phi = 62.7° \qquad \textit{Ans.}$$

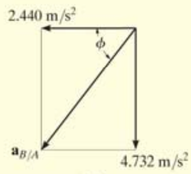


Fig. 12–45

NOTE: Is it possible to obtain the relative acceleration of $\mathbf{a}_{A/B}$ using this method? Refer to the comment made at the end of Example 12.26.